

# Blocking Artifacts Reduction in Block-based Dual-Tree Complex Wavelet Transform

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**Abstract**—We propose a new method to reduce blocking artifacts in an encoded image by block-based Dual-Tree Complex Wavelet. In this paper, we describe a cause of blocking artifacts of the block-based approach and focus on a method for finding sparse representations from redundant complex coefficients. The proposed method offers minimal transform operation performing on the same resolution for an input image. Experimental results show that the proposed method reduces blocking artifacts that are observed along block boundaries and thus improves the visual quality.

## I. INTRODUCTION

In block-based image or video processing, an image sequence is partitioned into blocks and encoded independently. In the standard video coding, motion estimation is performed with macro block basis. A suitable size of macro block achieves significant improvement in coding efficiency. In addition, a reduction of computation time can be achieved because block-based approach can be easily applied to parallel processing.

The main drawback of the block-based approach is blocking artifacts, which affect visual quality of encoded images. An image coding method using the discrete cosine transform, such as JPEG, suffers from blocking artifacts at low bitrates. To avoid the artifacts, the lapped orthogonal transform has been developed [1]. More recently, deblocking filters can be used for reducing the artifacts on block boundaries of video frames [2].

An alternative method for avoiding blocking artifacts in an encoded image is a wavelet approach. The wavelet transform does not suffer from the blocking artifacts because wavelet or scaling filters are applied throughout a whole image. Unfortunately, this advantage is only applicable to a full-frame compression. In practice, a tiling decomposition is performed for large images. When transform coefficients are quantized, the decomposition introduces artifacts [3]. Thus, designing of an appropriate filter is necessary to treat the boundary of images correctly [4]. In general, a reflexive boundary condition on the borders of the image is assumed for wavelet approach.

Recently, Dual-Tree Complex Wavelet (DTCWT) [5] has been developed as a new and powerful tool for analyzing signals. DTCWT is an overcomplete transform based on the discrete wavelet transform (DWT) and employs two real

DWTs to implement analytic wavelet transforms. Assuming that the implementation is block-based approach, encoded images suffer from blocking artifacts, which is similar to DWT. In addition, an operation to find sparse representations from DTCWT coefficients is a nonlinear system. Therefore, we can easily assume that the nonlinear operation is a cause of blocking artifacts.

In this paper, we describe the effect of nonlinear operation to the blocking artifacts and propose a simple solution to reduce the blocking artifacts in block-based DTCWT with coefficients optimization.

## II. DUAL-TREE COMPLEX WAVELET TRANSFORM FOR IMAGE CODING

In this section, we briefly describe the structure of DTCWT and optimization methods that select DTCWT coefficients to realize high coding efficiency.

### A. Dual-Tree Complex Wavelet Transform

DTCWT is a new transform based on overcomplete representation proposed by Kingsbury [5]. The main feature of DTCWT is that two DWT trees are performed in parallel. Each wavelet filter of the tree is designed to be Hilbert transform pair. A frequency response of the filter is given by

$$\mathcal{H}_\Omega = \begin{cases} -j, & \Omega > 0 \\ 0, & \Omega = 0 \\ j, & \Omega < 0. \end{cases} \quad (1)$$

If each filter satisfies (1), the coefficients can be formed as complex values. In a 2-D case, a set of six complex wavelets can be formed using tensor product of 1-D wavelets. Therefore, we can implement DTCWT without designing complicated filters. DTCWT is approximately shift invariant and offers higher directional selectivity. Subband coefficients of the six wavelets are shown in Fig. 1. A circle image is used as an input image. We see that each subband image represents a different directional singularity.

### B. Sparse Representation of DTCWT Coefficients

To obtain sparse representations from overcomplete transformed coefficients, several method has been developed [5]. An expansion of an image  $\mathbf{y}$  can be represented as

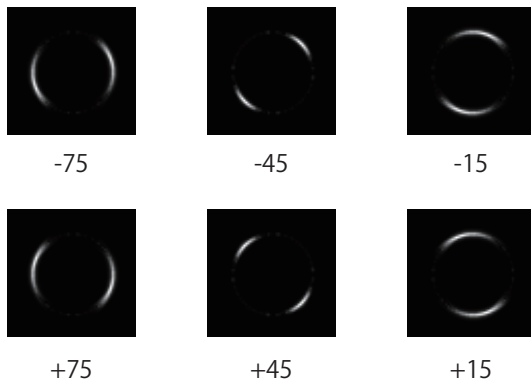


Fig. 1. Six subband images obtained by DTCWT.

$$\mathbf{y} = \Phi \mathbf{x}, \quad (2)$$

where  $\Phi$  is an  $M \times N$  matrix,  $\mathbf{x}$  is the coefficient vector of frame functions.  $\mathbf{x}_0 = \Psi \mathbf{y}$  is the minimum L2-norm solution and  $\Psi$  is the pseudo inverse of  $\Phi$ . Alternatively, the L0-norm solution is formulated as

$$\mathbf{x}_{L0} = \arg \min_{\mathbf{x}} \{\|\mathbf{x}\|_0 + \lambda \|\Phi \mathbf{x} - \mathbf{y}\|_2^2\}, \quad (3)$$

where  $\lambda \in \mathbb{R}^+$ . Mancera et al [6] have reformulated (3) as finding vector  $\mathbf{x}$  with  $K$  non-zero coefficients and minimizing the distance to  $\mathbf{y}$  concurrently. If  $C(K) = \{\mathbf{x} \in \mathbb{R}^M : \|\mathbf{x}\|_0 = K\}$ , then

$$\hat{\mathbf{x}}(K) = \arg \min_{\mathbf{x} \in C(K)} \|\Phi \mathbf{x} - \mathbf{y}\|_2^2. \quad (4)$$

An alternate projections method is utilized to resolve (4). The first set is  $C(K)$ . A vector  $\mathbf{x} \in C(K)$  has  $K$  non-zero coefficients. Note that this projection onto the set  $C(K)$  is nonlinear operation. The second is the affine set of solutions to (2). The iterative projections with the two sets are performed until the solution that satisfy an arbitrary norm is converged.

Reeves and Kingsbury [7] proposed a similar iterative method to find the optimal sparse representation of DTCWT coefficients. It is assumed that  $K$  increases gradually in the iterative operation. The combination of DTCWT and coefficients optimization method is applied to image coding [8]. A new method based on the combination outperforms the conventional DWT-based image coder. Fig. 2 shows an example, we could confirm its performance by the PSNR value around at 0.4[bpp].

### III. BLOCKING ARTIFACTS OF DTCWT

The cause of blocking artifacts in an encoded image are the filtering for an image boundary, quantization of coefficients, and nonlinear optimization.

We now focus on the filtering and quantization. The difference of filtering for an image boundary is shown in Fig. 3. An input image is generated by taking out the 350th horizontal line of Lena and then copying it 64 times in the vertical direction.

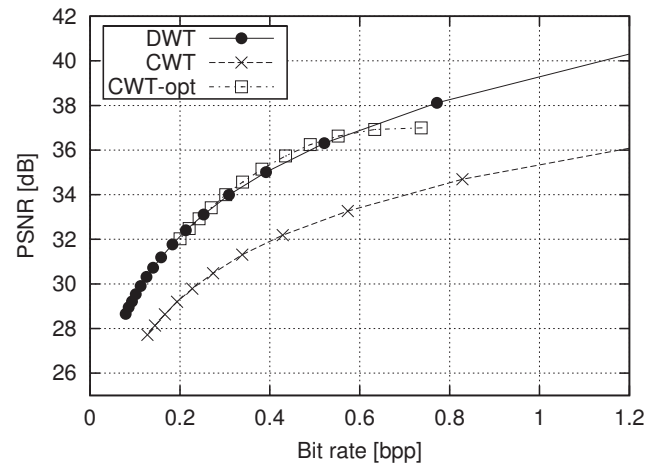


Fig. 2. Coding performance of DTWT with coefficients optimization.

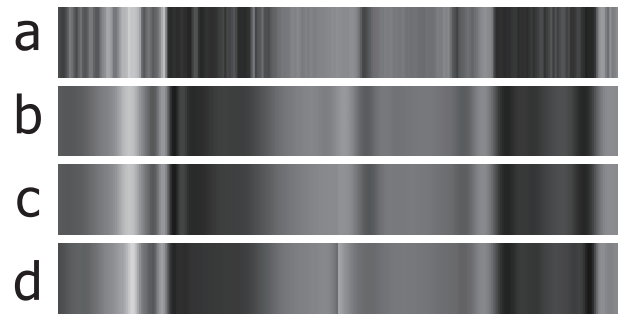


Fig. 3. Blocking artifacts at image boundary.

An original image is shown in Fig. 3(a). The result of filtering to the whole image and quantized is shown in Fig. 3(b). The result does not suffer from any blocking artifacts. Fig. 3(c) is a combined output image that is filtered independently with a mirroring signal extension. Fig. 3(d) is also a combined output that is filtered independently with a cyclic signal extension. The most noticeable artifacts can be seen in Fig. 3(d). Note that Fig. 3(c) also suffers from blocking artifacts.

We next discuss the nonlinear optimization of DTCWT coefficients. Since DTCWT is an overcomplete transform, the coefficients are four times redundant representation for 2-D images. Thus, as shown in the previous section, a coefficients optimization is necessary to achieve high coding efficiency. The output images processed using block-based DTCWT are shown in Fig. 4. The input image is Lena (512×512 pixels, grayscale), which is divided into 4 sub images and then each sub image is processed independently. Fig. 4 on the left shows the reconstructed image using the 2700 largest coefficients without optimization, while on the right shows the reconstructed image using the optimized 1000 sparse coefficients. PSNR is approximately the same value, which are about 31.1[dB]. We observe the blocking artifacts at the block borders in these two output images. Nevertheless, the artifacts in Fig. 4 on the right are more noticeable than the



Fig. 4. Blocking artifacts introduced by optimization.

left. This means that the  $M$  largest sparse coefficients have high frequency components to construct boundary edges. As a consequence, the iterative optimization is also the cause of blocking artifacts in block-based DTCWT.

#### IV. BLOCKING ARTIFACTS REDUCTION

In this section, we propose a new method to reduce the blocking artifacts introduced by coefficients optimization for DTCWT.

For implementing a simple block-based DTCWT, the input signal  $\mathbf{y} \in E^N$  is divided into sub blocks  $\mathbf{y}_i \in E^{N/b}$  for  $1 < i < b$  ( $i, b \in \mathbb{N}$ ) with proper block size.  $E^N$  is  $N$  dimensional Euclidean space and  $b$  is a number of sub blocks. Each sub block is transformed into coefficients using DTCWT independently. If  $\mathbf{x}_i = \Psi_i \mathbf{y}_i$ , then each vector  $\mathbf{x}_i \in E^{M/b}$  ( $M/b > N/b$ ) can be optimized by using iterative projections independently. Let  $\hat{\mathbf{x}}_i \in E^{M/b}$  be the optimal sparse representation of the transformed coefficients. Each vector  $\hat{\mathbf{x}}_i \in E^{M/b}$  is filtered by  $\Phi_i$ . Finally, the combination of  $\hat{\mathbf{y}}_i \in E^{N/b}$  in the same plane construct a full resolution output image  $\hat{\mathbf{y}} \in E^N$ . Unfortunately, as shown in the previous section, the output images produced by this simple procedure suffer from blocking artifacts.

To avoid the blocking artifacts, we use two different approaches in the parallel optimization procedure for sub blocks.

##### A. Initial vector for iterative projections

In general,  $\Psi_i \mathbf{y}_i (= \mathbf{x}_i)$  is used as the initial vector for each block in the iterative projections. The proposed parallel processing scheme is depicted in Fig. 5. In Fig. 5, the iterative projections is indicated as  $P_i$  for  $i$ th sub block. If the optimization process is performed at full resolution, the output of image does not suffer from blocking artifacts. Thus, the goal to find the sparse representation in each sub block is to obtain the same vector optimized at full resolution. In this scheme, the initial vector is  $\mathbf{x}_0 \in E^M$  for full resolution. Therefore, we divide  $\mathbf{x}_0$  into sub blocks  $\{\mathbf{x}_{0,i}\}$ , which is utilized as the initial vector for each sub block.

##### B. Feed back of a difference vector

An operator  $P_i$  for each sub block  $\{\mathbf{x}_i\}$  is implemented using filter banks with mirroring boundary extension. Thus, the combination of whole  $P_i$  in the same plane differs from  $P$  that is performed at full resolution. Nevertheless, coefficients

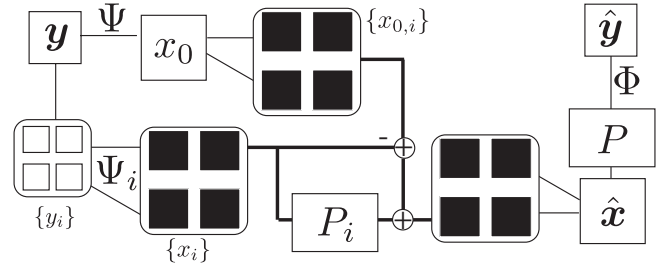


Fig. 5. The proposed parallel processing scheme.

optimization is processed with  $P_i$ . Thus, the converged vector never be the same one even though the initial vector is the same. To compensate this distance between converged vectors, we store the difference vector between  $\{\mathbf{x}_i\}$  and  $\{\mathbf{x}_{0,i}\}$ . After processing of the optimization, the difference vector is added to the output vector, which is optimized by  $P_i$ . This vector  $\hat{\mathbf{x}}$  is not a sparse representation because the difference vector is not sparse. Therefore, we utilize the optimization operator of  $P$  at full resolution only once to obtain the  $M$  largest significant coefficients. This optimization process also reduces the blocking artifacts of reconstructed images.

#### V. EXPERIMENTS

The proposed method is applied to standard images, Lena and Barbara. The input images are grayscale with size of  $512 \times 512$  pixels. The images are decomposed by 5-level DTCWT. We use 13-19 tap biorthogonal filters with mirroring boundary extension for the first stage of DTCWT. For the second and the following stages, 18 tap Q-shift orthogonal filters are used with cyclic boundary extension.

##### A. Sparse representation with the $M$ largest coefficients

We select the 8000 coefficients for Lena and the 12 000 largest coefficients for Barbara. The central regions cropped from the reconstructed images of Lena and Barbara are shown in Fig. 6 and Fig. 7.

The blocking artifacts on the face of Lena in Fig. 6(d) are reduced in comparison with that of Fig. 6(c). PSNR value of Fig. 6(c) and Fig. 6(d) are 34.66[dB] and 33.36[dB], respectively. The proposed method can reduce the blocking artifacts at the boundary of blocks. In Fig. 7(d), the blocking artifacts are reduced without losing texture information of Barbara.

##### B. Linear quantization of the $M$ largest coefficients

We obtain the optimized sparse DTCWT coefficients ( $M=12\ 000$ ) using the proposed method. The coefficients of DTCWT are quantized linearly to encode it at 0.2[bpp]. The reconstructed images with the conventional method and proposed method are shown in Fig. 8 and Fig. 9. Compared with the visual quality of the results, the significant improvement of reducing blocking artifacts can be verified.



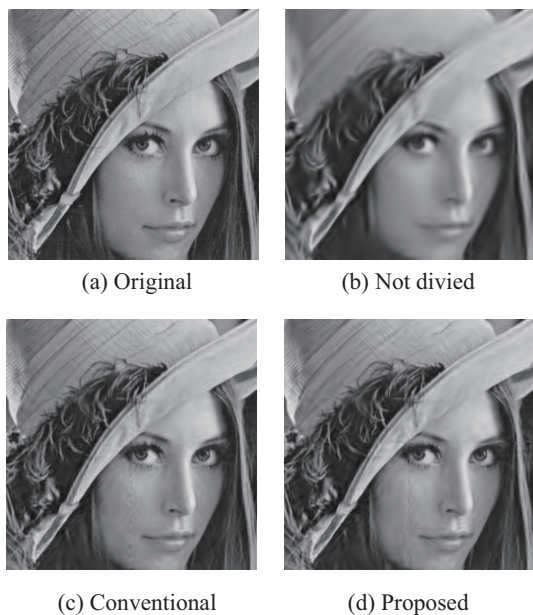


Fig. 6. Reconstructed images (Lena, M=8000).

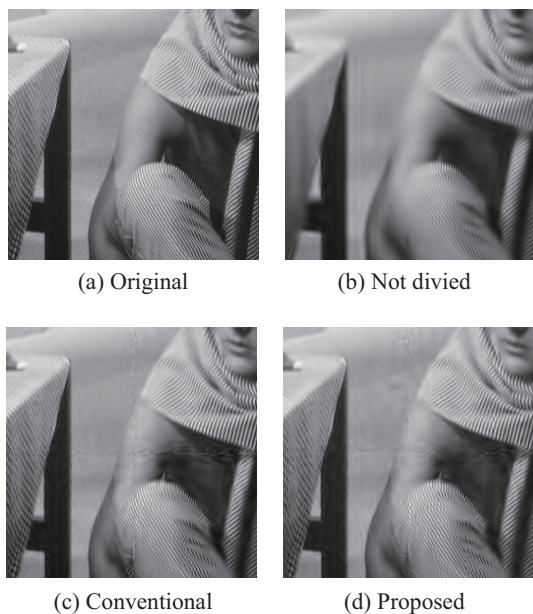


Fig. 7. Reconstructed images (Barbara, M=12 000).

### VI. CONCLUSIONS

We have presented a method that reduces blocking artifacts for block-based DTCWT. We have also discussed the cause of blocking artifacts. The proposed method is a simple solution to reduce the artifacts introduced by block-based iterative projections.

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Fig. 8. Conventional Method (0.2[bpp],31.30[dB]).



Fig. 9. Proposed Method (0.2[bpp],31.05[dB]).

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