# AUTOMATIC SCALE DETECTION BASED ON DIFFERENCE OF CURVATURE

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## ABSTRACT

Scale-invariant feature is an effective method for retrieving and classifying images. In this study, we analyze a scale-invariant planar curve features for developing 2D shapes. Scale-space filtering is used to determine contour structures on different scales. However, it is difficult to track significant points on different scales. In mathematics, curvature is considered to be fundamental feature of a planar curve. However, the curvature of a digitized planar curve depends on a scale. Therefore, automatic scale detection for curvature analysis is required for practical use. We propose a technique for achieving automatic scale detection based on difference of curvature. Once the curvature values are normalized with regard to the scale, we can calculate difference in the curvature values for different scales. Further, an appropriate scale and its position are detected simultaneously, thereby avoiding tracking problem. Appropriate scales and their positions can be detected with high accuracy. An advantage of the proposed method is that the detected significant points do not need to be located in the same contour. The validity of the proposed method is confirmed by experimental results.

**Keywords:** scale detection, curvature, planar curve, pattern recognition

## 1. INTRODUCTION

Scale-invariant feature is an effective method for retrieving and classifying images. For example, scale-invariant feature transform (SIFT) features are invariant to image scaling and rotation [1]. The SIFT algorithm is widely used for achieving content-based image retrieval (CBIR) and solving image classification problems. For example, in mathematics, curvature is considered to be a fundamental feature of a planar curve, and it is invariant to a coordinate system. In this study, we analyze 2D shapes such as planar curves on the basis of their curvature.

The curvature of a continuous planar curve is unrelated to a scale because the differential of the curve is uniquely determined. However, the curvature of a discrete planar curve depends on the scale, because the differential of a discrete signal processing has a degree of freedom. For example, the differential of the discrete signal is defined as the convolution between Gaussian derivative filter and the discrete signal. Furthermore, significant points of planar curve vary on a coarse-to-fine viewpoint. Therefore, it is important to consider the scale while determining the curvature.

Thus far, curvature has been considered to be an important parameter for developing a large number of shape descriptors in the field of pattern recognition and computer vision. In the early stage, scale-space filtering for developing 1D planar curve was proposed [2]. Although this technique has been effective in determining the structure of the planar curve, tracking and describing significant points has seemed to be difficult. Later, a contour-based shape descriptor based on the curvature scale space (CSS) representation of the contour has been realized [3]. This representation was further extended and optimized during the MPEG development phase.

However, shape descriptors based on the CSS do not detect the appropriate scale and its position. Tracking of significant points in CSS representation of the contour is still needed. The SIFT algorithm solves the above similar problems using the difference of Gaussian, which approximates scale-normalized Laplacian of Gaussian. Therefore, we propose a technique for achieving automatic scale detection which detects the appropriate scale and its position on the basis of difference of curvature (DoC) without tracking them. Instead of Gaussian in the DoG, a scale-normalized curvature is employed for the DoC. The detection of the appropriate scale and its position is termed “scale point.” An advantage of scale point is that feature points do not need to be in a single shape.

In the following section, an overview of the SIFT algorithm and the CSS representation is presented. Then, the automatic scale detection method is described in detail. Finally, some experimental results are shown.

## 2. RELATED WORKS

### 2.1 SIFT [1]

The SIFT algorithm is used to detect and describe the local gradient features of images [1]. The gradient of the SIFT image determines the features, which are invariant to image scaling and rotation. The SIFT algorithm uses an extrema of the scale-normalized Laplacian of Gaussian (sLoG) for detecting appropriate scale.

Since the computational cost of the sLoG is high, it is approximated by the DoG such as

$$\mathrm{sLoG} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k - 1}, \tag{1}$$

where $k$ is the ratio of adjacent scales. Further, when the image is down-sampled, the computational cost of DoG is dramatically reduced. The detected appropriate scale is used to normalize the significant points (keypoints).

### 2.2 Curvature Scale Space

The curvature of planar curve is invariant to the coordinate system. The curvature $\kappa$ of the planar curve represented by a parametric equation $(x(t), y(t))$ is defined by

$$\kappa = \frac{x'y'' - x''y'}{(x'^2 + y'^2)^{3/2}}. \tag{2}$$

Here, $x$ and $y$ are differentiated with respect to parameter $t$.

Scale-space filtering has been proposed to analyze the structure of planar curve [2]. The planar curve is convoluted with Gaussian filters at different scales resulting in the formation of smoothed curves. An extrema of the first order differential of the curve or the zero-crossing point of the second order differential of the curve are considered to be the feature points.

For example, the designed of the contour-based shape descriptor is based on the CSS representation of the contour [3]. In the past, this representation has been successfully used to search and retrieve, and it has been further extended and optimized during the MPEG development phase.

To develop a contour-shaped CSS descriptor, $N$ equidistant points are selected on the contour. Then, the contour is gradually smoothed by the repeted application of a low-pass filter with a kernel $(0.25, 0.5, 0.25)$ to the $X$ and $Y$ coordinates of the selected $N$ contour points. The horizontal coordinates of the CSS image correspond to the indices of the contour points which are selected to represent the contour $(1, \ldots, N)$, and the vertical coordinates of the CSS image correspond to the amount of filtering carried out which is defined as the number of passes of the filter. For each smoothed contour, the zero-crossing of its curvature function is computed. The CSS image has characteristic peaks. The coordinate values of the prominent peaks of the CSS image are determined. A sample of the CSS image is shown in Fig. 1, while a "driver" as an input shape is shown in Fig. 2. In Fig. 2, the center of the circle indicates the position of the significant points, and the radius of the circle indicates the number of passes of the filter, which are nonlinearly transformed.

However, there are three problems in the CSS representation. First, rigid tracking of feature points scattered in various smoothed curves is difficult. In practice, rough pattern matching is sufficient. Second, the number of passes of the filter does not correspond to appropriate scale. Third the relationship between significant points in different shapes is not considered.

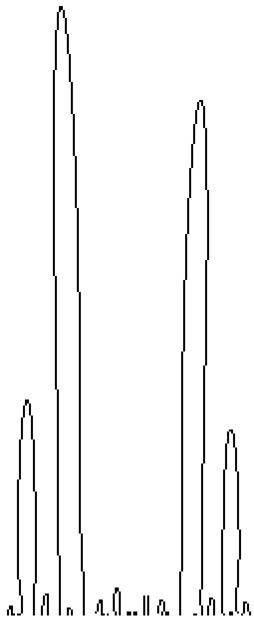

Fig. 1: CSS of the "driver."

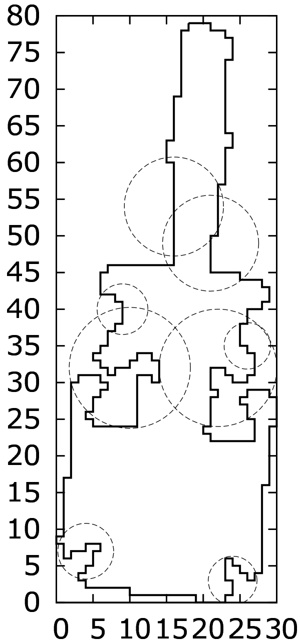


Fig. 2: Significant points with scales of the "driver."

## 3. PROPOSED METHOD

### 3.1 Scale-normalized Curvature

The differential of a discrete parametric curve should be defined on a scale. It implies that differential of a discrete parametric curve is defined by a convolution of a derivative Gaussian function with a parametric curve. The Gaussian function is defined by

$$G(t, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{t^2}{2\sigma^2}). \tag{3}$$

This function is derived using the standard deviation. In this work, standard deviation is termed scale. The differential of the discrete parametric curve is also derived using the scale. Therefore the curvature of this curve is also derived by using the scale. Hence, the differential is defined

$$x' = x'[t, \sigma] = G'[t, \sigma] * x[t], \tag{4}$$
$$y' = y'[t, \sigma] = G'[t, \sigma] * y[t]. \tag{5}$$

The curvatures of various scales should be normalized. We consider a curvature operator such as

$$\frac{G_x' G_y'' - G_x'' G_y'}{(G_x'^2 + G_y'^2)^{3/2}}. \tag{6}$$

This operator is used in the case of a parametric curve. Now, we simplify the operator in order to define a scale normalize factor. We assume that $x[t] = t$, $x' = 1$, and $x'' = 0$. Therefore, the curvature operator can be simplified to

$$\frac{G''}{(1 + G'^2)^{3/2}}. \tag{7}$$

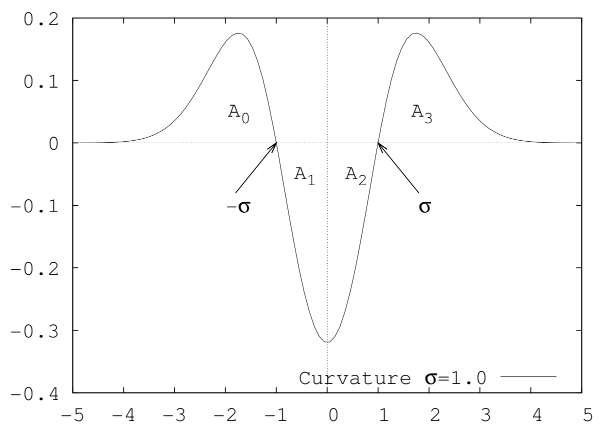


Fig. 3: Shape of curvature operator.

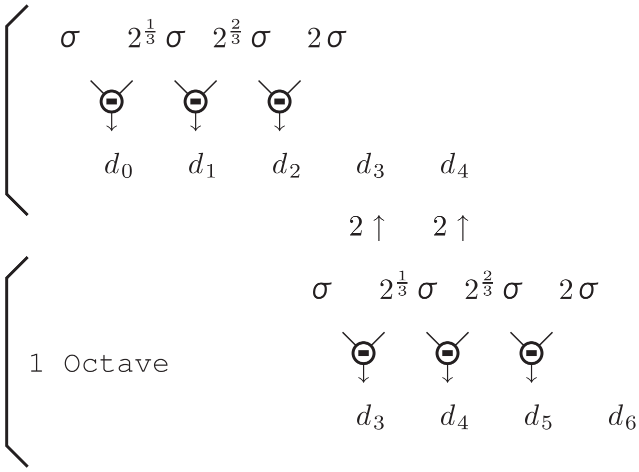


Fig. 4: Octave structure resulting from down-sampling.

The shape of this operator is shown in Fig. 3. In this figure, the four areas $(A_0, A_1, A_2, A_3)$ have the same size. Then, the scale factor of this area is chosen as a standard. The scale factor is defined such that the area is always constant on any scale. The area is calculated by

$$A_2 = \int_0^{\sigma} \frac{-G''(t,\sigma)}{(1+G'(t,\sigma)^2)^{3/2}} \mathrm{d}t = \frac{1}{\sqrt{1+2\pi e\sigma^4}}. \quad (8)$$

Therefore, the scale factor is $\sqrt{1+2\pi e\sigma^4}$. In the following section, we use a scale-normalized curvature, because we calculate the difference in the value of the curvatures on varying scales.

### 3.2 Difference of Curvature

The scale-normalized curvature has been defined using the scale. Therefore, instead of Gaussian in the DoG, a scale-normalized curvature is employed for the DoC.

In this section, we describe the use of DoC to detect the scale of curvature. The scales of several scale-normalized curvatures are determined. The ratio of adjacent scales is the same. The difference in the value of the curvatures on adjacent scales at same point is estimated. Local maximum/minimum values simultaneously define the scale point. The accuracy of detecting the scale point is improved by interpolating a quadratic function. Finally, an accurate scale point is automatically detected without tracking.

### 3.3 Down-sampling

In order to obtain a large scale, the tap length of the scale-normalized curvature operator should be long. To avoid an increase in the computational cost, we reduce the tap length of a Gaussian derivative filter, which is responsible for the formation of the curvature.

The composition of the Gaussian filter is described. The convolution of two Gaussian filters comprising $\sigma_1$ and $\sigma_2$ with the parametric curve could be achieved by convoluting one Gaussian function comprising $\sqrt{\sigma_1^2+\sigma_2^2}$ with the parametric curve. This rule is easily applied to the derivative Gaussian filter. Furthermore, smoothing the curve using the Gaussian filter comprising $2\sigma$ and down-sampling it is equal to down-sampling the curve and smoothing it using the Gaussian filter comprising $\sigma$. This rule limits the expansion of the scale. Down-sampling is carried out for the sufficiently smoothed planar curve.

For the detection of appropriate scales, three DoCs required and four scale-normalized curvatures are required. They should consist of same sampling points. The number of points in some cases is half because of down-sampling. Therefore, linear interpolation can be used to up-sampling. An octave structure, resulting from the down-sampling of the curve and DoCs, is shown in Fig. 4.

If a small number of samples are used, the accuracy of detecting of the position of the scale point is lower. It is improved by interpolating a quadratic function.

### 3.4 Refinement of scale points

The detected feature points are often redundant, i.e., some feature points are located on adjacent place where the scales are different. In fact, several appropriate scales may exist for a partial shape. Therefore, the scale points are refined if necessary.

Scale points are ordered by a decreasing scale. If a scale point of small scale exists on the adjacent location of one of large scale, this scale point of small scale is removed. The threshold distance between two scale points is larger in proportion to the scale.

## 4. EXPERIMENTAL RESULTS

### 4.1 Scale Invariance

The validity of the DoC is confirmed by an experimental result. Two shapes are developed where the size of one is double that of the other. The result of two scale points is shown in Fig. 5 to Fig. 6. The horizontal axis indicates the scales, while the vertical axis indicates the value of DoC.

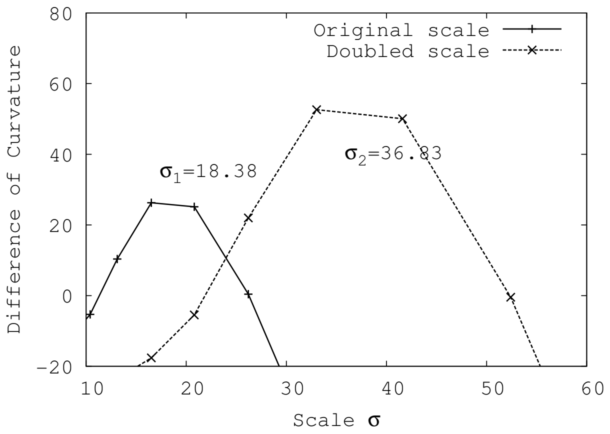


Fig. 5: Relation between scales and DoCs for two shapes on the point 1.

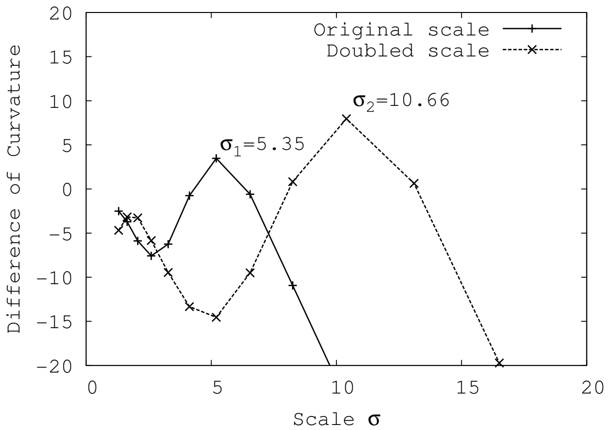


Fig. 6: Relation between scales and DoCs for two shapes on the point 2.

From these figures, it can be observed that the detected scales are double such as two shapes. Other scale points are also the same relation.

### 4.2 Influence of Down-sampling

Although it is confirmed that automatic scale detection is realized by performing some preliminary experiments, the scale points are moved from the position where they are detected by the DoC without down-sampling. The reason of the movement is that the curvature of the shape is sensitive to the change of shape. The results of the two DoCs, i.e., DoC with/without down-sampling, are shown in Figs. 7 and 8.

Therefore, down-sampling of the curve during DoC is more difficult compared to that during SIFT. In the rest of this study, we show the results of DoC without down-sampling.

### 4.3 Comparison of DoC and CSS

The scale points detected by the proposed DoC and the conventional CSS, which are superimposed on the original con-

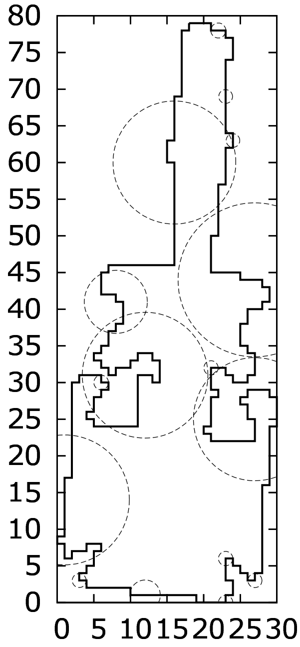

Fig. 7: DoC of "driver" with down sampling.

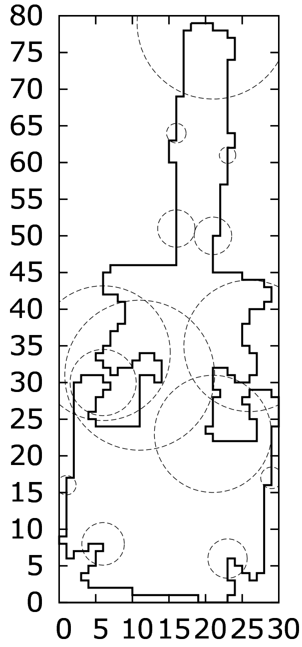

Fig. 8: DoC of "driver" without down sampling.

tour, are shown in Figs. 9 and 24, respectively. These shapes are part of the Part B of CE-Shape-1 [4]. In these figures, the radiuses of the circles are in proportion to the detected scale or the number of passes of filter.

The distribution of the scale points detected by the DoC tends to differ from those detected by the CSS. The proposed DoC algorithm is used to detect convex shapes of intermediate size. This is because that the DoC algorithm focuses in the change of curvature. In contrasts the CSS algorithm, it used to detect features of shapes. Note that the curvature-based feature detection techniques are not effective for detecting large convex. For example, Figs. 9 and 10 do not have any feature points except a stalk of apples.

An advantage of the DoC algorithm is the ability to detect local scale points. Therefore, a scale point is considered to be standard when a part of shape is dramatically changed. Further, the relative position of the scale points is determined by normalizing the scale. The scale points do not need to be located in the same contour.

## 5. CONCLUSIONS

In this study, we propose a technique for achieving automatic scale detection for planar curves based on difference of curvature. The detection of appropriate scale and its position, which is termed "scale point," is analogous to the difference of Gaussian (DoG) in the SIFT algorithm. Therefore, the concept of a scale-normalized curvature and difference of curvature is introduced. The accuracy of detected the scale point is improved by interpolating quadratic functions. The scale invariance of the scale point is shown in some experimental results. Therefore, we conclude that the scale point is a scale-invariant feature.

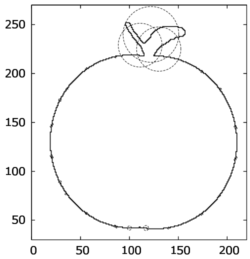


Fig. 9: DoC of “apple.”

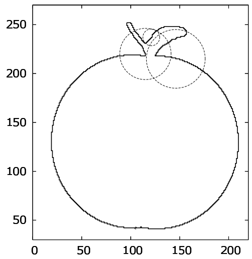


Fig. 10: CSS of “apple.”

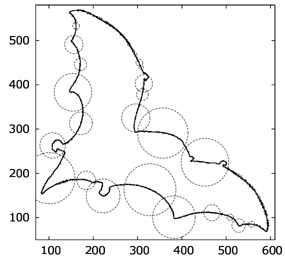


Fig. 11: DoC of “bat.”

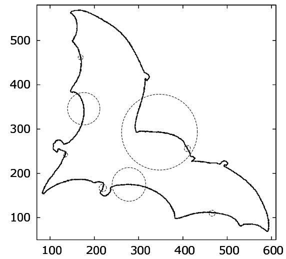


Fig. 12: CSS of “bat.”

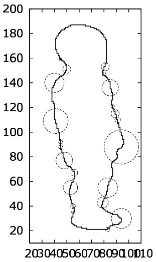


Fig. 13: DoC of “children.”

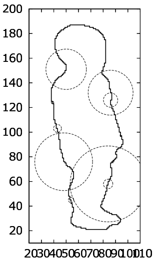


Fig. 14: CSS of “children.”

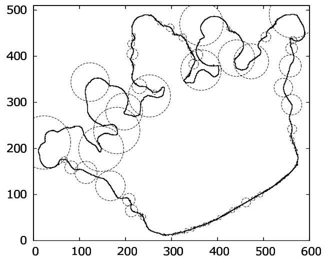


Fig. 15: DoC of “crown.”

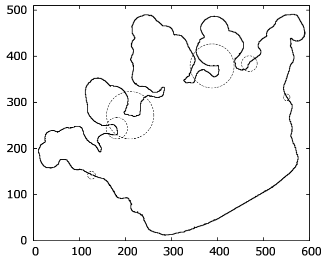


Fig. 16: CSS of “crown.”

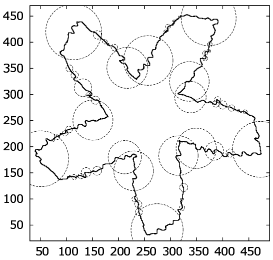


Fig. 17: DoC of “device0.”

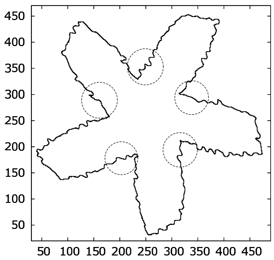


Fig. 18: CSS of “device0.”

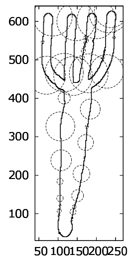


Fig. 19: DoC of “fork.”

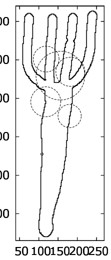


Fig. 20: CSS of “fork.”

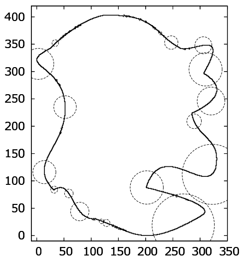


Fig. 21: DoC of “Misk.”

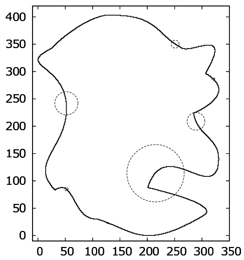


Fig. 22: CSS of “Misk.”

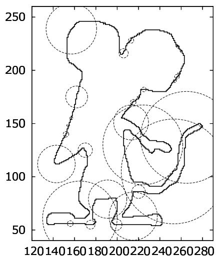


Fig. 23: DoC of “rat.”

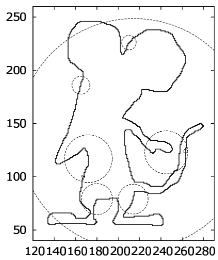


Fig. 24: CSS of “rat.”

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