マルチメディア配信システム

- No.3 画像圧縮技術 (1) -

渡辺　裕

Multimedia Distribution System

- No.3 Image Compression Technology (1) -

Hiroshi Watanabe

画像圧縮技術

- 色信号多重分離
- 直交変換
- サブバンド
- ウェーブレット
- 係数量子化
- エントロピー符号化

Image Compression Technology

- Chrominance Signal Mux/DeMux
- Orthogonal Transform
- Subband
- Wavelet
- Coefficient Quantization
- Entropy Coding

色信号処理

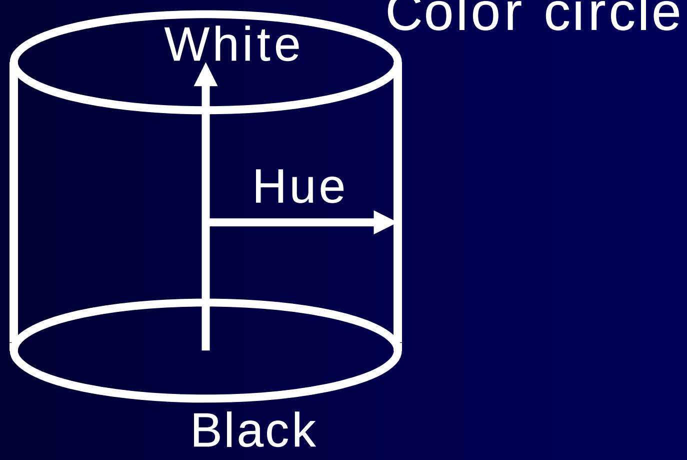
- カラーテレビジョン信号
  - 輝度信号に色信号を高調波として多重化
  - YIQ (Y:Luminance, I,Q:Chrominance)
- カラー静止画像
  - 光の3原色
  - RGB (R:Red, G:Green, B:Blue)
- CCIR勧告601 (ITU-R 601)
  - YCbCr (Y:Luminance, Cb,Cr:Chrominance)

Color Signal Processing

- Color Television Signal
  - Multiplexing chrominance signal to luminance signal by modulation
  - YIQ (Y:Luminance, I,Q:Chrominance)
- Color Still Picture
  - three primary colors for light
  - RGB (R:Red, G:Green, B:Blue)
- CCIR Recomm.601 (ITU-R 601)
  - YCbCr (Y:Luminance, Cb,Cr:Chrominance)

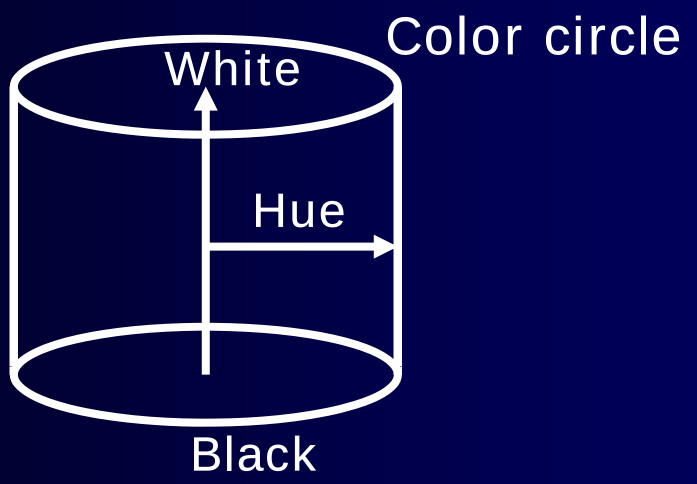
色信号処理 (2)

- 印刷
  - YMCK (Y:Yellow, M:Magenta, C:Cyan, K:Black)
- その他
  - CIE XYZ, CIE L*a*b*



Color Signal Processing (2)

- Printing
  - CMYK (C:Cyan, M:Magenta, Y:Yellow, K:Black)
- Others
  - CIE XYZ, CIE L*a*b*



信号変換

- RGB to YCbCr

$$Y = 0.229R + 0.587G + 0.114B$$
$$Cb = -0.170R - 0.331G + 0.500B + 128$$
$$Cr = 0.500R - 0.419G - 0.081B + 128$$

- YCbCr to RGB

$$R = Y + 1.402(Cr - 128)$$
$$G = Y - 0.344(Cb - 128) - 0.714(Cr - 128)$$
$$B = Y + 1.772(Cb - 128)$$

Signal Conversion

- RGB to YCbCr

$$Y = 0.229R + 0.587G + 0.114B$$

$$Cb = -0.170R - 0.331G + 0.500B + 128$$

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$$R = Y + 1.402(Cr - 128)$$

$$G = Y - 0.344(Cb - 128) - 0.714(Cr - 128)$$

$$B = Y + 1.772(Cb - 128)$$

信号変換 (2)

- RGB to YIQ

$$Y = 0.30R + 0.59G + 0.11B$$
$$I = 0.60R - 0.28G - 0.32B$$
$$Q = 0.21R - 0.52G - 0.31B$$

- YIQ to RGB

$$R = Y + 0.96I + 0.62Q$$
$$G = Y - 0.27I - 0.65Q$$
$$B = Y - 0.10I + 1.70Q$$

Signal Conversion (2)

- RGB to YIQ

$$Y = 0.30R + 0.59G + 0.11B$$
$$I = 0.60R - 0.28G - 0.32B$$
$$Q = 0.21R - 0.52G - 0.31B$$

- YIQ to RGB

$$R = Y + 0.96I + 0.62Q$$
$$G = Y - 0.27I - 0.65Q$$
$$B = Y - 0.10I + 1.70Q$$

信号変換 (3)

- RGB to CIE XYZ

$$X = 0.6070R + 0.1740G + 0.2000B$$
$$Y = 0.2990R + 0.5870G + 0.1140B$$
$$Z = 0.0000R + 0.0660G + 1.1110B$$

- CIE XYZ to CIE L*a*b*

$$L^* = 116(Y/Y_n)^{1/3} - 16 \quad \text{for} \quad Y/Y_n > 0.008856$$
$$L^* = 903.3(Y/Y_n) \quad \text{for} \quad Y/Y_n \le 0.008856$$
$$a^* = 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}]$$
$$b^* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}]$$

Signal Conversion (3)

- RGB to CIE XYZ

$$X = 0.6070R + 0.1740G + 0.2000B$$
$$Y = 0.2990R + 0.5870G + 0.1140B$$
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$$b^* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}]$$

HVS

- HVS (Human Vision System)
  - 色変化より輝度変化に敏感
  - 色差信号のサンプリング点数を削減
  - YCbCr形式においてCbCrを水平垂直にサブサンプルした例

RGB

Y

CbCr

HVS

- HVS (Human Vision System)
  - Sensitive to luminance change than chrominance
  - Down sampling to chrominance signal
  - Example of vertical and horizontal sub-sampling of CbCr at YCbCr format

RGB

Y

CbCr

カラーテレビジョン信号

- アナログ形式
  - NTSC信号
  - PAL信号
  - SECAM信号

- ディジタル形式
  - RGB (4:4:4)
  - 4:2:2形式　および　そのファミリー
  - CIF, QCIF
  - SIF, QSIF

Color Television Signal

- Analog Format
  - NTSC Signal
  - PAL Signal
  - SECAM Signal
- Digital Format
  - RGB (4:4:4)
  - 4:2:2 Format and it’s family
  - CIF, QCIF
  - SIF, QSIF

NTSC信号

- NTSC (National Television Systems Committee)
- 米国, カナダ, 日本, 韓国
- フレーム数 29.97 frame/sec (59.94 field/sec)
- ライン数　525 line/frame
- 水平同期周波数 fh＝15.7342KHz
- 周波数帯域 4.2MHz (I:1.5MHz, Q:0.5MHz)
- 色搬送波 (455/2)fh＝3.58MHz
- アスペクト比　4:3

NTSC Signal

- NTSC (National Television Systems Committee)
- US, Canada, Japan, Korea
- Frames 29.97 frame/sec (59.94 field/sec)
- Lines 525 line/frame
- Horizontal Sync. Frequency fh=15.7342KHz
- Bandwidth 4.2MHz (I:1.5MHz, Q:0.5MHz)
- Chrominance Carry Wave (455/2) fh=3.58MHz
- Aspect Ratio 4:3

PAL信号

- PAL (Phase Alternating Line)
- ヨーロッパ, 中南米, アフリカ, アジア
- フレーム数 25 frame/sec (50 field/sec)
- ライン数 625 line/frame
- 水平同期周波数 fh＝15.625KHz
- 周波数帯域 5.0MHz
- 色搬送波 4.43MHz

PAL Signal

- PAL (Phase Alternating Line)
- Europe, Latin America, Africa, Asia
- Frames 25 frame/sec (50 field/sec)
- Lines 625 line/frame
- Horizontal Sync. Frequency fh=15.625KHz
- Bandwidth 5.0MHz
- Chrominance Carry Wave 4.43MHz

SECAM信号

- SECAM (Systeme Electronique Couleur Avec Memoire)
- フランス, ロシア
- 基本的にPALと解像度が同じ
- 色信号の多重化方法がPALと異なる

SECAM Signal

- SECAM (Systeme Electronique Couleur Avec Memoire)
- France, Russia
- Basic resolution is the same as PAL’s one
- Mux system of chrominance signal is different from PAL

ITU-R勧告601

- ITU-R(当時はCCIR)から勧告されたディジタルTVフォーマット
- カラーTVの分離符号化が目的
- NTSC系とPAL系に共通のサンプリング周波数を設定(13.5MHz)
- 輝度と色差信号のサンプリング周波数の比率を，4:4:4と表し，ディジタルフォーマットファミリーを形成
- 量子化は 8 bit/pel
- Y信号は 16(black) to 235 (white)
- CbCr信号は 128 を中心として peak to peak 255

ITU-R Recommendation 601

- Digital TV format recommended by ITU-R (used be CCIR)
- Color separation coding of TV signal
- Set common sampling frequency for NTSC and PAL system (13.5MHz)
- Ratio of sampling frequency for luminance and chrominance is noted as 4:4:4, form digital format family
- Quantization: 8 bit/pel
- Dynamic range of luminance: 16(black) to 235 (white)
- CbCr signal: Center is 128, peak to peak 255

4:2:2 Format Family

- 4:2:2
  - 4:2:2スタジオフォーマットあるいはD1フォーマットと呼ばれる
  - YCrCb信号のサンプリング周波数はそれぞれ，13.5MHz，6.75MHz
  - NTSC系では，Y:720x486，CbCr:360x486
  - PAL系では，Y:720x576，CbCr:360x576
  - NTSC系では486ラインのうち480ラインが有効走査線
- 4:2:0
  - 色差信号のどちらかのサンプリングが走査線毎に交番
  - JPEG，MPEGにおいて主に使用される.
  - NTSC系では，Y:720x480，CbCr:360x240
  - PAL系では，Y:720x576，CbCr:360x288

4:2:2 Format Family

- 4:2:2
  - 4:2:2 Studio format or called D1 format
  - Sampling frequency for YCrCb signals are 13.5MHz, 6.75MHz
  - NTSC area, Y:720x486, CbCr:360x486
  - PAL area, Y:720x576, CbCr:360x576
  - 480 of 486 lines are effective in NTSC area
- 4:2:0
  - Alternate sampling of chrominance signals
  - Used for JPEG, MPEG
  - NTSC area, Y:720x480, CbCr:360x240
  - PAL area, Y:720x576, CbCr:360x288

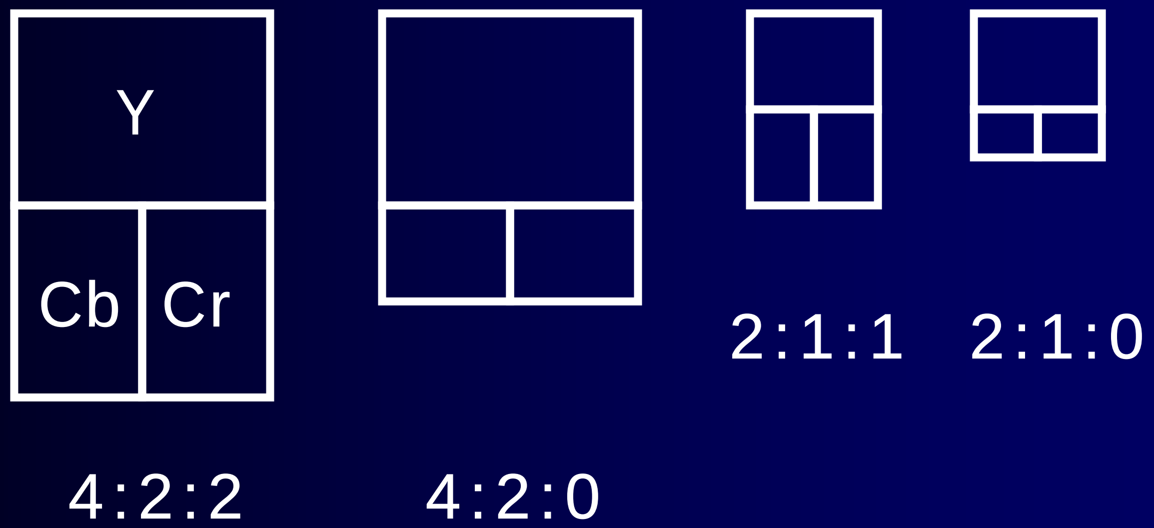
4:2:2 Format Family (2)

- 2:1:1
  - 4:2:2に比べて，水平垂直に1/2にサブサンプリングされたフォーマット
  - NTSC系では， Y:360x240, CbCr:180x240
  - PAL系では, Y:360x288, CbCr:180x288
- 2:1:0
  - 4:2:0に比べて，水平垂直に1/2にサブサンプリングされたフォーマット
  - NTSC系では， Y:360x240, CbCr:180x120
  - PAL系では, Y: 360x288, CbCr:180x144

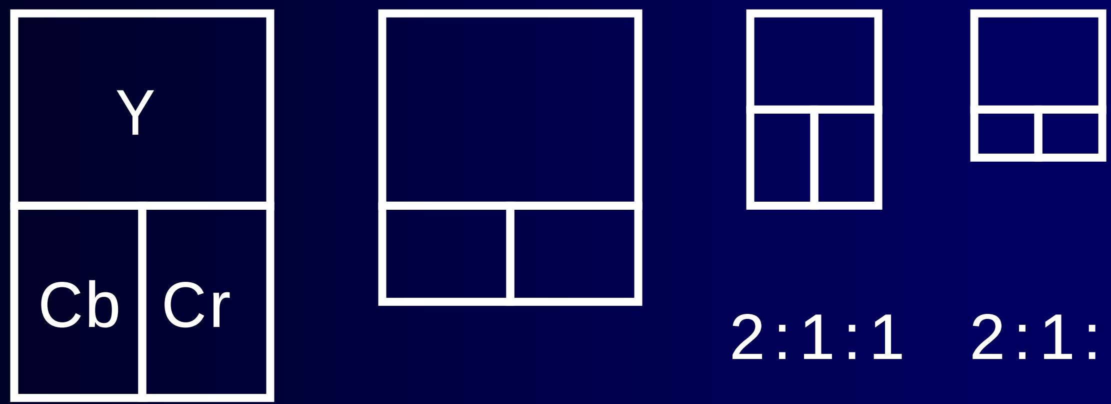
4:2:2 Format Family (2)

- 2:1:1
  - 1/2 sub-sampled horizontally and vertically from 4:2:2
  - NTSC area, Y:360x240, CbCr:180x240
  - PAL area, Y:360x288, CbCr:180x288
- 2:1:0
  - 1/2 sub-sampled horizontally and vertically from 4:2:0
  - NTSC area, Y:360x240, CbCr:180x120
  - PAL area, Y: 360x288, CbCr:180x144

4:2:2 Format Family (3)



4:2:2 Format Family (3)



4:2:2 4:2:0 2:1:1 2:1:0

画像圧縮技術

- 色信号多重分離
- 直交変換
- サブバンド
- ウェーブレット
- 係数量子化
- エントロピー符号化

Image Compression Technology

- Chrominance Signal Mux/DeMux
- Orthogonal Transform
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直交変換

- 離散フーリエ変換
- アダマール変換
- KL変換
- 離散サイン変換
- 離散コサイン変換

Orthogonal Transform

- Discrete Fourier Transform
- Hadamard Transform
- KL Transform
- Discrete Sine Transform
- Discrete Cosine Transform

離散フーリエ変換

- Ｎ点離散フーリエ変換と逆変換

$$X(k)=\sum_{n=0}^{N-1}x(n)\exp(-j2\pi nk/N)$$

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X(k)\exp(j2\pi nk/N)$$

離散的な角周波数

$$\omega_k=\frac{2\pi k}{N}\quad(k=0,1,\cdots,N-1)$$

Discrete Fourier Transform

- N-point Discrete Fourier Transform and Inverse transform

$$X(k) = \sum_{n=0}^{N-1} x(n) \exp(-j2\pi nk / N)$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp(j2\pi nk / N)$$

Discrete frequency $\omega_k = \dfrac{2\pi k}{N} \quad (k = 0, 1, \cdots, N-1)$

離散フーリエ変換の特徴

- 離散フーリエ変換係数
  - N点の整数値からN点の複素数への変換
  - 変換係数はN個の実数と虚数
  - 符号化すべきデータ個数は2倍に増加し、データ圧縮には不適
  - 直流成分は係数1個で表現される

Characteristics of DFT

- DFT coefficients
  - From N-point integer to N-point complex value
  - Coefficients are N real numbers and N imaginary numbers
  - the number of data for coding application increased to 2 times, thus not suitable for data compression
  - DC component can be represented by one coefficient

アダマール変換

- 変換マトリクスの要素が *[+1、-1]* からなる最も簡単な直交変換
- ハードウエア化が容易
- *2n* 次の変換マトリクスは、*2x2(n=1)* の基本マトリクスから拡張

$$\begin{bmatrix} X(0) \\ X(1) \end{bmatrix} = \boldsymbol{H}_2 \begin{bmatrix} x(0) \\ x(1) \end{bmatrix}$$

$$\boldsymbol{H}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- 拡張規則

$$\boldsymbol{H}_{2n} = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{H}_n & \boldsymbol{H}_n \\ \boldsymbol{H}_n & -\boldsymbol{H}_n \end{bmatrix}$$

Hadamard Transform

- Element of transform matrix are *[+1、-1]*
- Simple orthogonal matrix, easy hardware realization
- $2n$-th transform matrix can be derived from $2x2(n=1)$-th basic matrix

$$\begin{bmatrix} X(0) \\ X(1) \end{bmatrix} = \boldsymbol{H}_2 \begin{bmatrix} x(0) \\ x(1) \end{bmatrix}$$

$$\boldsymbol{H}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- Expansion rule

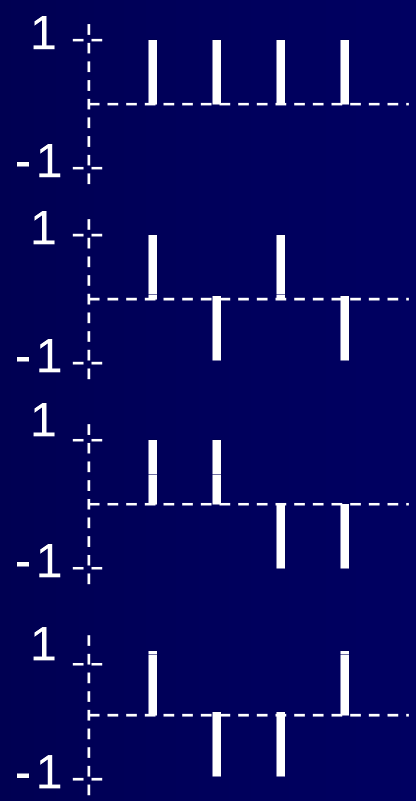
$$\boldsymbol{H}_{2n} = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{H}_n & \boldsymbol{H}_n \\ \boldsymbol{H}_n & -\boldsymbol{H}_n \end{bmatrix}$$

アダマール変換(2)

- H_4, H_8

$$\boldsymbol{H}_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\boldsymbol{H}_8 = \frac{1}{\sqrt{8}}\begin{bmatrix} \boldsymbol{h(0)}^t \\ \boldsymbol{h(1)}^t \\ \vdots \\ \boldsymbol{h(7)}^t \end{bmatrix}$$

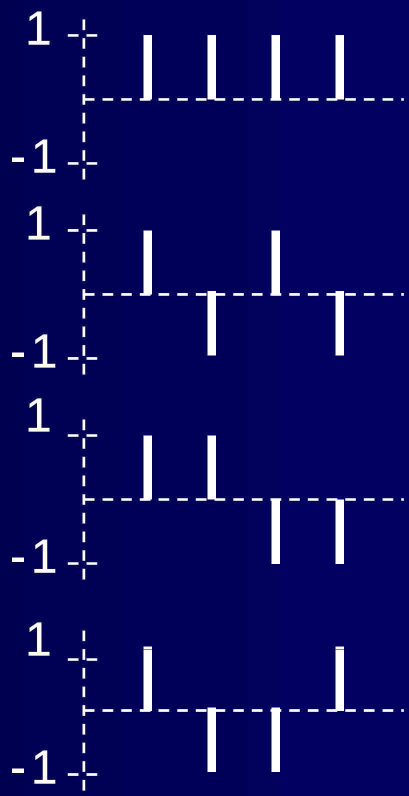


Hadamard Transform(2)

- H_4, H_8

$$H_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$H_8 = \frac{1}{\sqrt{8}}\begin{bmatrix} h(0)^t \\ h(1)^t \\ \vdots \\ h(7)^t \end{bmatrix}$$



アダマール変換(3)

- H_8 のベクトルの要素

$$\boldsymbol{h(0)}^t = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$\boldsymbol{h(1)}^t = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$
$$\boldsymbol{h(2)}^t = \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{bmatrix}$$
$$\boldsymbol{h(3)}^t = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$
$$\boldsymbol{h(4)}^t = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}$$
$$\boldsymbol{h(5)}^t = \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \end{bmatrix}$$
$$\boldsymbol{h(6)}^t = \begin{bmatrix} 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$
$$\boldsymbol{h(7)}^t = \begin{bmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

Hadamard Transform(3)

- vector element of H_8

$$
\begin{aligned}
h(0)^t &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \\
h(1)^t &= \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix} \\
h(2)^t &= \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{bmatrix} \\
h(3)^t &= \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix} \\
h(4)^t &= \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix} \\
h(5)^t &= \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \end{bmatrix} \\
h(6)^t &= \begin{bmatrix} 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix} \\
h(7)^t &= \begin{bmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}
\end{aligned}
$$

ウォルシュ変換

- W_8 の変換マトリクスの要素は
  アダマール変換の順序を変えたもの

$$W_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} h(0)^t \\ h(4)^t \\ h(6)^t \\ h(2)^t \\ h(3)^t \\ h(7)^t \\ h(5)^t \\ h(1)^t \end{bmatrix}$$

Walsh Transform

- Elements of Walsh Transform W_8 are represented by different order of Hadamard Transform's elements

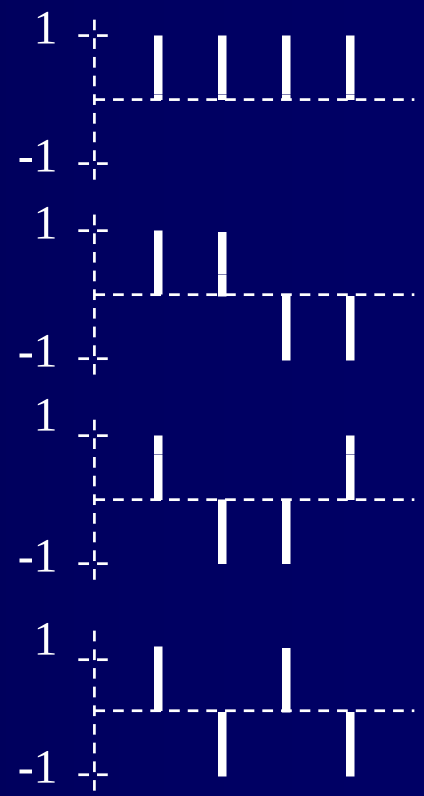
$$W_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} h(0)^t \\ h(4)^t \\ h(6)^t \\ h(2)^t \\ h(3)^t \\ h(7)^t \\ h(5)^t \\ h(1)^t \end{bmatrix}$$

ウォルシュ変換の例

- W_4 による変換の例

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} x(0)+x(1)+x(2)+x(3) \\ x(0)+x(1)-x(2)-x(3) \\ x(0)-x(1)-x(2)+x(3) \\ x(0)-x(1)+x(2)-x(3) \end{bmatrix}$$

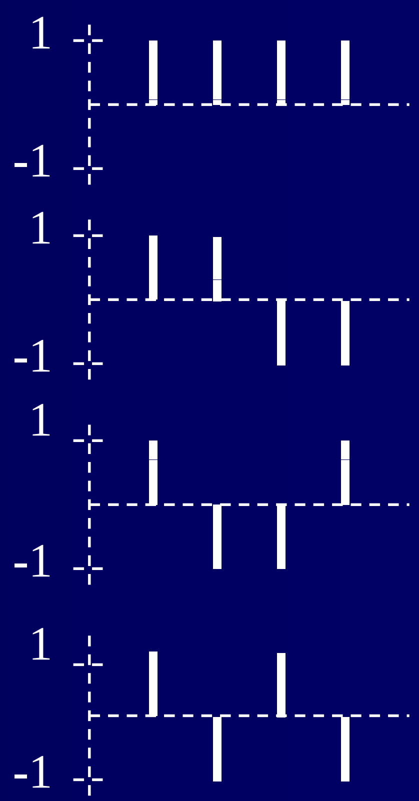


Example of Walsh Transform

- Example of Transform by W_4

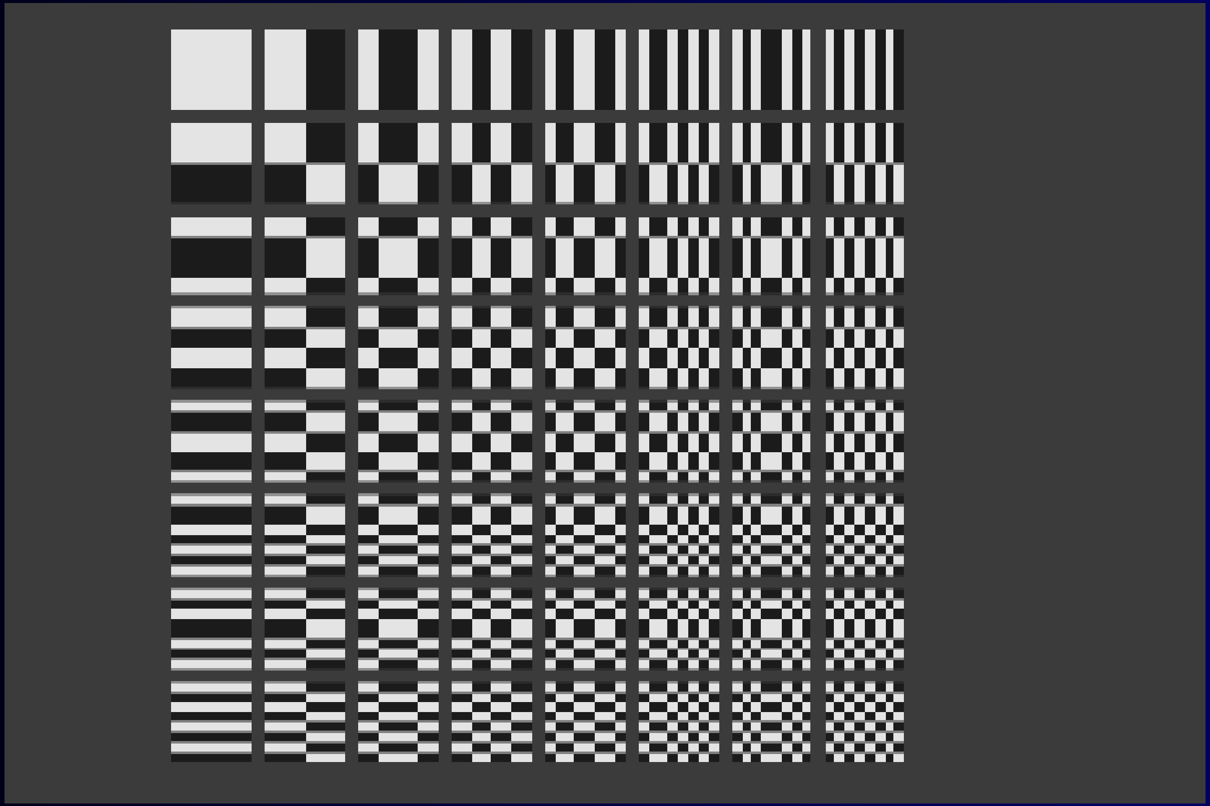
$$
\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}
$$

$$
= \frac{1}{2} \begin{bmatrix} x(0)+x(1)+x(2)+x(3) \\ x(0)+x(1)-x(2)-x(3) \\ x(0)-x(1)-x(2)+x(3) \\ x(0)-x(1)+x(2)-x(3) \end{bmatrix}
$$



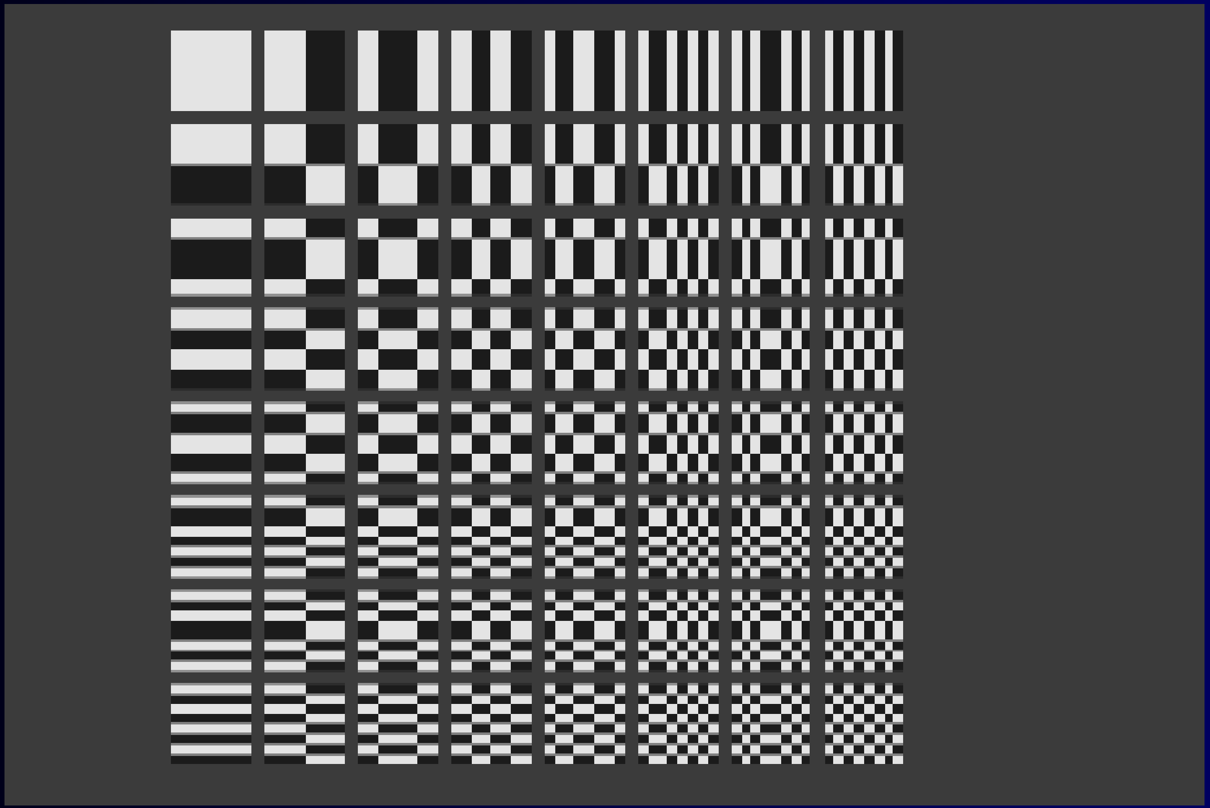
基底画像

- 8x8ウォルシュ変換の基底画像



Picture of Walsh Basis

- Picture of 8x8 Walsh Transform basis



アダマール変換の特徴

- 構成が簡単
- 基底の形状が階段状であり、なだらかな波形の近似に向かない
- 信号の圧縮に用いると、階段状（ブロック状）の雑音を生じる
- 圧縮効率は低い

Characteristics of Hadamard Transform

- Very simple structure
- Shape of basis is step like, thus difficult to approximate smoothly changing signal
- Cause step like (block) noise for signal compression
- Low compression efficiency

KL(Karhunen-Loeve)変換

- 入力信号 $x(n)$ の自己相関行列 R_{xx} を対角化
- 対角化の際の固有ベクトルを基底ベクトルとする変換

$$
\mathbf{R_{xx}} = E\left[\begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} \begin{bmatrix} x(0) & x(1) & \cdots & x(N-1) \end{bmatrix} \right]
$$

$$
= \begin{bmatrix} r(0) & r(1) & \cdots & r(N-1) \\ r(1) & r(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & r(1) \\ r(N-1) & \cdots & r(1) & r(0) \end{bmatrix}
$$

KL(Karhunen-Loeve) Transform

- Diagonalize auto-correlation matrix R_{xx} of input signal $x(n)$
- Basis vector of transform is eigenvector

$$\mathbf{R_{xx}} = E\left[\begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}\begin{bmatrix} x(0) & x(1) & \cdots & x(N-1) \end{bmatrix}\right]$$

$$= \begin{bmatrix} r(0) & r(1) & \cdots & r(N-1) \\ r(1) & r(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & r(1) \\ r(N-1) & \cdots & r(1) & r(0) \end{bmatrix}$$

KL(Karhunen-Loeve)変換(2)

- 固有ベクトル t_n を要素とする行列 T による対角化

$$\boldsymbol{T}\,\boldsymbol{R}_{xx}\,\boldsymbol{T}^{t} = \begin{bmatrix} \sigma_0^2 & 0 & \cdots & 0 \\ 0 & \sigma_1^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{N-1}^2 \end{bmatrix}$$

$$\boldsymbol{T}^{t} = \begin{bmatrix} t_0(0) & t_1(0) & \cdots & t_{N-1}(0) \\ t_0(1) & t_1(1) & \cdots & t_{N-1}(1) \\ \vdots & \vdots & \cdots & \vdots \\ t_0(N-1) & t_1(N-1) & \cdots & t_{N-1}(N-1) \end{bmatrix}$$

KL(Karhunen-Loeve)Transform(2)

- Diagonalize by matrix T which elements are eigenvector t_n

$$\boldsymbol{T}\,\boldsymbol{R}_{xx}\,\boldsymbol{T}^t = \begin{bmatrix} \sigma_0^2 & 0 & \cdots & 0 \\ 0 & \sigma_1^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{N-1}^2 \end{bmatrix}$$

$$\boldsymbol{T}^t = \begin{bmatrix} t_0(0) & t_1(0) & \cdots & t_{N-1}(0) \\ t_0(1) & t_1(1) & \cdots & t_{N-1}(1) \\ \vdots & \vdots & \cdots & \vdots \\ t_0(N-1) & t_1(N-1) & \cdots & t_{N-1}(N-1) \end{bmatrix}$$

KL(Karhunen-Loeve)変換(3)

- KL変換係数 $k(n)$ $(n=0,\ldots, N-1)$ は入力信号を $x(n)$ $(n=0,\ldots,N-1)$ とすると次式で与えられる

$$
\begin{bmatrix} k(0) \\ k(1) \\ \vdots \\ k(N-1) \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}
$$

$$
= \begin{bmatrix} t_0(0) & t_0(1) & \cdots & t_0(N-1) \\ t_1(0) & t_1(1) & \cdots & t_1(N-1) \\ \cdots & \cdots & \cdots & \cdots \\ t_{N-1}(0) & t_{N-1}(1) & \cdots & t_{N-1}(N-1) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}
$$

KL(Karhunen-Loeve)Transform(3)

- KL transform coefficient $k(n)$ $(n=0,\ldots,N\text{-}1)$ for input signal $x(n)$ $(n=0,\ldots,N\text{-}1)$ can be obtained by

$$\begin{bmatrix} k(0) \\ k(1) \\ \vdots \\ k(N-1) \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

$$= \begin{bmatrix} t_0(0) & t_0(1) & \cdots & t_0(N-1) \\ t_1(0) & t_1(1) & \cdots & t_1(N-1) \\ \cdots & \cdots & \cdots & \cdots \\ t_{N-1}(0) & t_{N-1}(1) & \cdots & t_{N-1}(N-1) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

KL変換係数

- KL変換係数は互いに無相関

$$\boldsymbol{k} = \boldsymbol{T}\boldsymbol{x}$$

$$\begin{aligned}
E[\mathbf{k}\mathbf{k}^t] &= E[\mathbf{T}\mathbf{x}(\mathbf{T}\mathbf{x})^t] \\
&= E[\mathbf{T}\mathbf{x}\mathbf{x}^t\mathbf{T}^t] \\
&= \mathbf{T}E[\mathbf{x}\mathbf{x}^t]\mathbf{T}^t \\
&= \mathbf{T}\mathbf{R}_{\mathbf{xx}}\mathbf{T}^t \\
&= diag\begin{bmatrix}\sigma_0^2 & \sigma_1^2 & \cdots & \sigma_{N-1}^2\end{bmatrix}
\end{aligned}$$

KL Transform Coefficient

- KL Transform coefficients are not correlated each other

$$\boldsymbol{k} = \boldsymbol{T}\boldsymbol{x}$$

$$\begin{aligned}
E[\mathbf{k}\mathbf{k}^t] &= E[\mathbf{T}\mathbf{x}(\mathbf{T}\mathbf{x})^t] \\
&= E[\mathbf{T}\mathbf{x}\mathbf{x}^t\mathbf{T}^t] \\
&= \mathbf{T}E[\mathbf{x}\mathbf{x}^t]\mathbf{T}^t \\
&= \mathbf{T}\mathbf{R}_{\mathbf{xx}}\mathbf{T}^t \\
&= diag\begin{bmatrix}\sigma_0^2 & \sigma_1^2 & \cdots & \sigma_{N-1}^2\end{bmatrix}
\end{aligned}$$

KL変換係数(2)

- 無相関とは...

$$E[k(n)k(m)] \begin{cases} =0 & (n \neq m) \\ \neq 0 & (n = m) \end{cases}$$

$$E[\mathbf{kk}^t] = \begin{bmatrix} E[k(0)k(0)] & E[k(0)k(1)] & \cdots & E[k(0)k(N-1)] \\ E[k(1)k(0)] & E[k(1)k(1)] & \cdots & E[k(1)k(N-1)] \\ \vdots & \vdots & \ddots & \vdots \\ E[k(N-1)k(0)] & E[k(N-1)k(1)] & \cdots & E[k(N-1)k(N-1)] \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_0^2 & 0 & \cdots & 0 \\ 0 & \sigma_1^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{N-1}^2 \end{bmatrix}$$

KL Transform Coefficient(2)

- What is “not correlated”?

$$E[k(n)k(m)] \begin{cases} = 0 & (n \neq m) \\ \neq 0 & (n = m) \end{cases}$$

$$E[\mathbf{kk}^t] = \begin{bmatrix} E[k(0)k(0)] & E[k(0)k(1)] & \cdots & E[k(0)k(N-1)] \\ E[k(1)k(0)] & E[k(1)k(1)] & \cdots & E[k(1)k(N-1)] \\ \vdots & \vdots & \ddots & \vdots \\ E[k(N-1)k(0)] & E[k(N-1)k(1)] & \cdots & E[k(N-1)k(N-1)] \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_0^2 & 0 & \cdots & 0 \\ 0 & \sigma_1^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{N-1}^2 \end{bmatrix}$$

復習

- 行列 A の対角化には固有ベクトル t が必要

$$\boldsymbol{At} = \lambda \boldsymbol{t}$$

- どうやって固有値λを求めるか？ 固有方程式を計算する.

$$(\lambda \boldsymbol{I} - \boldsymbol{A})\boldsymbol{t} = 0$$

- 固有値は，左辺の行列式を解いて得る

$$\left| \lambda \boldsymbol{I} - \boldsymbol{A} \right| = 0$$

- 対角化する行列は固有ベクトルを並べたもの

$$\boldsymbol{T} = [\, \boldsymbol{t}_0 \quad \boldsymbol{t}_1 \quad \cdots \quad \boldsymbol{t}_{N-1} \,]$$

Review

- We need eigenvector t to diagonalize matrix A

$$\boldsymbol{At} = \lambda \boldsymbol{t}$$

- How to obtain eigenvalue l? Characteristic equation.

$$(\lambda \boldsymbol{I} - \boldsymbol{A})\boldsymbol{t} = 0$$

- Eigenvalue can be obtained by solving determinant of the left term

$$|\lambda \boldsymbol{I} - \boldsymbol{A}| = 0$$

- Matrix for diagonalization is a series of eigenvector

$$\boldsymbol{T} = [\boldsymbol{t}_0 \quad \boldsymbol{t}_1 \quad \cdots \quad \boldsymbol{t}_{N-1}]$$

KL変換の特徴

- 低次のシーケンシーに対するパワー集中は理論的に最高
- 入力信号の性質によって相関行列が異なる
- そのため、入力信号ごとにKL変換基底が異なる
- 受信側に基底ベクトルを送信する必要がある
- 理論限界を与えるものであり、実用的ではない

Characteristics of KL Transform

- Theoretically best performance to compact energy to the low sequencies
- Autocorrelation matrix differs depends on input signal
- Thus, KL transform basis differs depends on each input
- Needs to transmit basis vector to the receiver
- Gives theoretical bound, not practical

離散サイン変換

- N点離散サイン変換(DST-II)と逆変換

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=1}^{N}x(n)\sin\left(\frac{(2n-1)k\pi}{2N}\right)\quad(k=1,\ldots,N)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{k=1}^{N}C(k)X(k)\sin\left(\frac{(2n-1)k\pi}{2N}\right)\quad(n=1,\ldots,N)$$

ここに

$$C(k)=\begin{cases}\dfrac{1}{\sqrt{2}} & (k=N)\\ 1 & (k\neq N)\end{cases}$$

Discrete Sine Transform

- N-point Discrete Sine Transform(DST-II) and Inverse DST

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=1}^{N}x(n)\sin\left(\frac{(2n-1)k\pi}{2N}\right)\quad(k=1,\ldots,N)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{k=1}^{N}C(k)X(k)\sin\left(\frac{(2n-1)k\pi}{2N}\right)\quad(n=1,\ldots,N)$$

where

$$C(k)=\begin{cases}\dfrac{1}{\sqrt{2}} & (k=N)\\ 1 & (k\neq N)\end{cases}$$

離散サイン変換(2)

- DST-I

$$\sin\left(\frac{nk\pi}{N}\right) \quad (k,n=1,2,\ldots,N-1)$$

- DST-II

$$C(k)\sin\left(\frac{(2n-1)k\pi}{2N}\right) \quad (k,n=1,2,\ldots,N)$$

- DST-III

$$C(n)\sin\left(\frac{(2k-1)n\pi}{2N}\right) \quad (k,n=1,2,\ldots,N)$$

- DST-IV

$$\sin\left(\frac{(2k+1)(2n+1)\pi}{4N}\right) \quad (k,n=0,1,\ldots,N-1)$$

Discrete Sine Transform(2)

- DST-I

$$\sin\left(\frac{nk\pi}{N}\right) \quad (k,n=1,2,\ldots,N-1)$$

- DST-II

$$C(k)\sin\left(\frac{(2n-1)k\pi}{2N}\right) \quad (k,n=1,2,\ldots,N)$$

- DST-III

$$C(n)\sin\left(\frac{(2k-1)n\pi}{2N}\right) \quad (k,n=1,2,\ldots,N)$$

- DST-IV

$$\sin\left(\frac{(2k+1)(2n+1)\pi}{4N}\right) \quad (k,n=0,1,\ldots,N-1)$$

離散サイン変換(3)

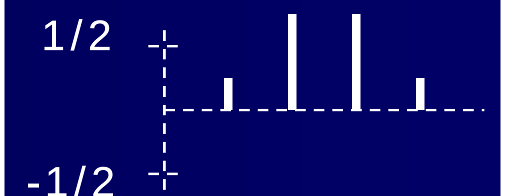
- 第1係数の計算

$$X(1)=\sqrt{\frac{2}{N}}C(1)\sum_{n=1}^{N}x(n)\sin\left(\frac{(2n-1)\pi}{2N}\right)$$

- 4点DSTの場合の第1番目の基底

$$\sqrt{\frac{2}{N}}C(1)\sin\left(\frac{(2n-1)\pi}{2N}\right) \qquad (n=1,2,3,4)$$

$$=\sqrt{\frac{1}{2}}\left[\sin\frac{\pi}{8} \quad \sin\frac{3\pi}{8} \quad \sin\frac{3\pi}{8} \quad \sin\frac{\pi}{8}\right]$$



Discrete Sine Transform(3)

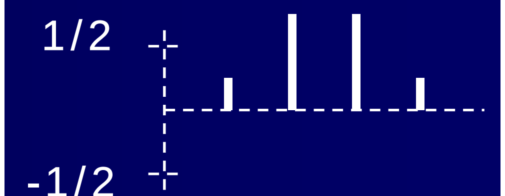
- Calculation of the first coefficient

$$X(1)=\sqrt{\frac{2}{N}}C(1)\sum_{n=1}^{N}x(n)\sin\left(\frac{(2n-1)\pi}{2N}\right)$$

- The first basis of 4-point DST

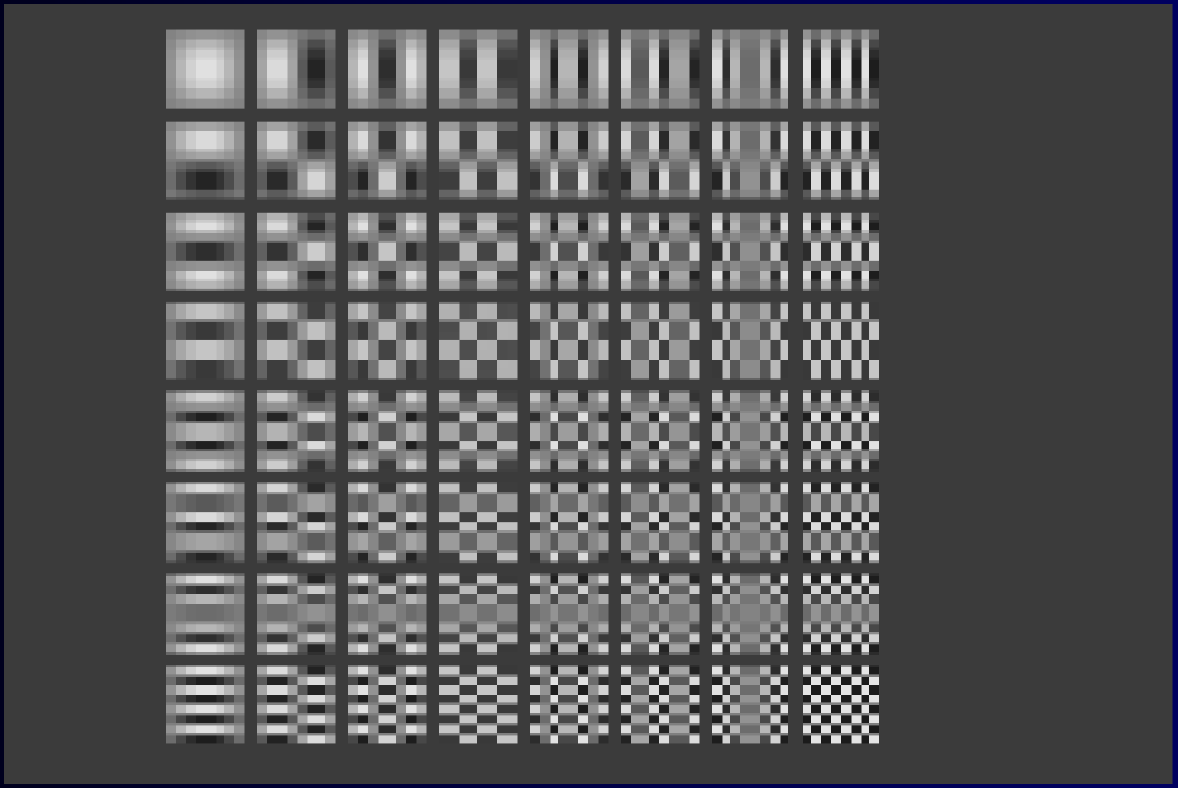
$$\sqrt{\frac{2}{N}}C(1)\sin\left(\frac{(2n-1)\pi}{2N}\right)\quad (n=1,2,3,4)$$

$$=\sqrt{\frac{1}{2}}\left[\sin\frac{\pi}{8}\quad \sin\frac{3\pi}{8}\quad \sin\frac{3\pi}{8}\quad \sin\frac{\pi}{8}\right]$$



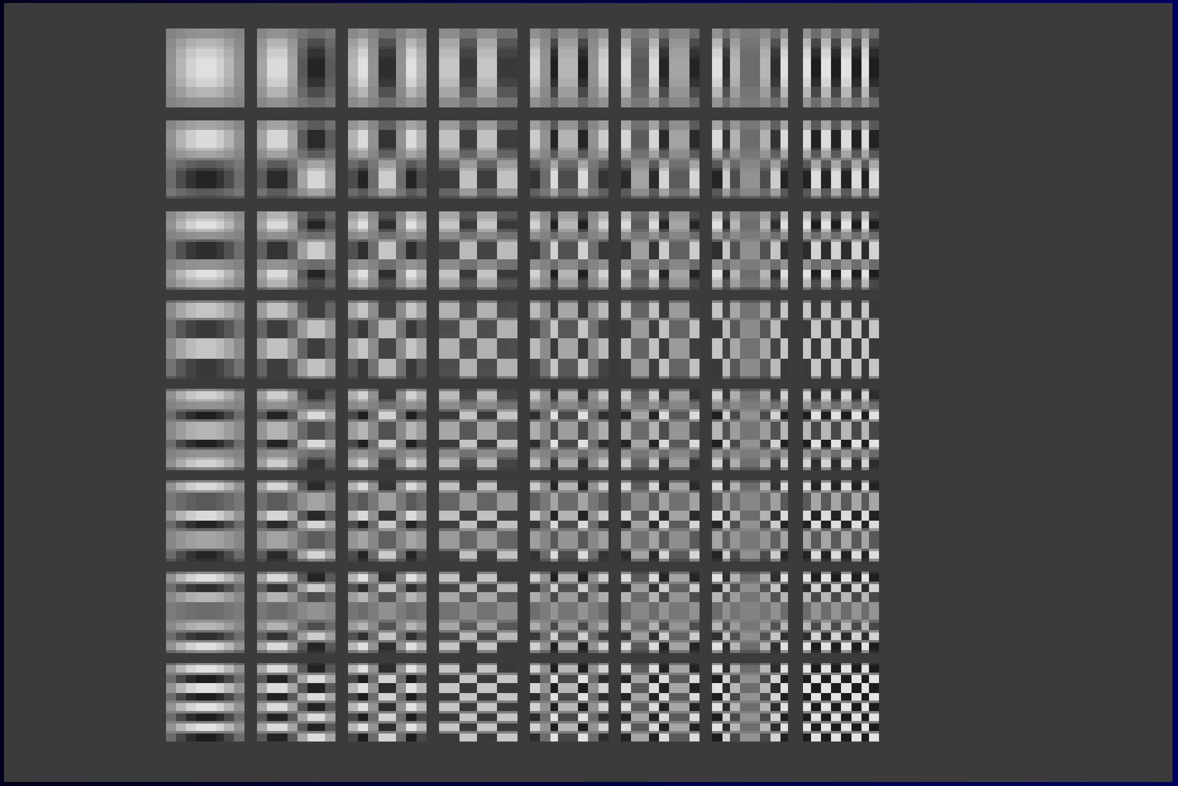
基底画像

- 8x8DSTの基底画像



Picture of DST Basis

- Picture of 8x8 DST basis



離散サイン変換の特徴

- 直流成分を1個の変換係数で表現できない欠点がある
- 基底ベクトルの値は、原理的に区間の端点で0
- 隣接区間とのデータの連続性は良い

Characteristics of DST

- Cannot represent DC component by one coefficient
- Edge value of basis vector is zero
- Smooth connection of data with the neighbor section

離散コサイン変換 (DCT)

- N点DCT(タイプII) ... 信号 $x(n)$, 係数 $X(k)$

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=0}^{N-1}x(n)\cos\left(\frac{(2n+1)k\pi}{2N}\right)\quad (k=0,\ldots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{u=0}^{N-1}C(k)X(k)\cos\left(\frac{(2n+1)k\pi}{2N}\right)\quad (n=0,\ldots,N-1)$$

ここに,

$$C(k)=\begin{cases}\dfrac{1}{\sqrt{2}} & (k=0)\\ 1 & (k\neq 0)\end{cases}$$

Discrete Cosine Transform (DCT)

- N point DCT(Type-II) ... Signal *x(n)*, Coefficient *X(k)*

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=0}^{N-1}x(n)\cos\left(\frac{(2n+1)k\pi}{2N}\right)\quad (k=0,\dots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{u=0}^{N-1}C(k)X(k)\cos\left(\frac{(2n+1)k\pi}{2N}\right)\quad (n=0,\dots,N-1)$$

where,

$$C(k)=\begin{cases}\frac{1}{\sqrt{2}} & (k=0)\\ 1 & (k\neq 0)\end{cases}$$

離散コサイン変換(2)

- DCT-I

$$C(k)C(n)\cos\left(\frac{nk\pi}{N}\right) \quad (k,n=0,1,\ldots,N)$$

- DCT-II

$$C(k)\cos\left(\frac{(2n+1)k\pi}{2N}\right) \quad (k,n=0,1,\ldots,N-1)$$

- DCT-III

$$C(n)\cos\left(\frac{(2k+1)n\pi}{2N}\right) \quad (k,n=0,1,\ldots,N-1)$$

- DCT-IV

$$\cos\left(\frac{(2n+1)(2k+1)\pi}{4N}\right) \quad (k,n=0,1,\ldots,N-1)$$

Discrete Cosine Transform(2)

- DCT-I

$$C(k)C(n)\cos\left(\frac{nk\pi}{N}\right) \quad (k,n=0,1,\ldots,N)$$

- DCT-II

$$C(k)\cos\left(\frac{(2n+1)k\pi}{2N}\right) \quad (k,n=0,1,\ldots,N-1)$$

- DCT-III

$$C(n)\cos\left(\frac{(2k+1)n\pi}{2N}\right) \quad (k,n=0,1,\ldots,N-1)$$

- DCT-IV

$$\cos\left(\frac{(2n+1)(2k+1)\pi}{4N}\right) \quad (k,n=0,1,\ldots,N-1)$$

2次元DCT

- NxN点2次元DCT ... 信号 *x(n,m)*, 係数 *X(u,v)*

$$X(u,v)=\frac{2}{N}C(u)C(v)\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}x(n,m)\cos\left(\frac{(2n+1)u\pi}{2N}\right)\cos\left(\frac{(2m+1)v\pi}{2N}\right)$$

$$x(n,m)=\frac{2}{N}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}C(u)C(v)X(u,v)\cos\left(\frac{(2n+1)u\pi}{2N}\right)\cos\left(\frac{(2m+1)v\pi}{2N}\right)$$

ここに,

$$C(u),C(v)=\begin{cases}\frac{1}{\sqrt{2}} & (u=0,v=0)\\ 1 & (u\neq 0,v\neq 0)\end{cases}$$

Two-Dimensional DCT

- NxN point 2-D DCT ... Signal *x(n,m)*, Coefficient *X(u,v)*

$$X(u,v)=\frac{2}{N}C(u)C(v)\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}x(n,m)\cos\left(\frac{(2n+1)u\pi}{2N}\right)\cos\left(\frac{(2m+1)v\pi}{2N}\right)$$

$$x(n,m)=\frac{2}{N}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}C(u)C(v)X(u,v)\cos\left(\frac{(2n+1)u\pi}{2N}\right)\cos\left(\frac{(2m+1)v\pi}{2N}\right)$$

where,

$$C(u),C(v)=\begin{cases}\frac{1}{\sqrt{2}} & (u=0,v=0)\\ 1 & (u\neq 0,v\neq 0)\end{cases}$$

2次元DCTの計算

- 変数 *n, m* に関して独立な1次元DCTの処理に分解
- 2回のN点DCT演算として計算できる

$$t(u,m)=\sqrt{\frac{2}{N}}C(u)\sum_{n=0}^{N-1}x(n,m)\cos\left(\frac{(2n+1)u\pi}{2N}\right)$$

$$X(u,v)=\sqrt{\frac{2}{N}}C(v)\sum_{m=0}^{N-1}t(u,m)\cos\left(\frac{(2m+1)v\pi}{2N}\right)$$

Calculation of 2-D DCT

- Decompose to independent 1-D DCT with regard to variables *n, m*
- Calculated as 2 times N point DCT

$$t(u,m) = \sqrt{\frac{2}{N}}C(u)\sum_{n=0}^{N-1} x(n,m)\cos\left(\frac{(2n+1)u\pi}{2N}\right)$$

$$X(u,v) = \sqrt{\frac{2}{N}}C(v)\sum_{m=0}^{N-1} t(u,m)\cos\left(\frac{(2m+1)v\pi}{2N}\right)$$

8x8DCT

- 8x8点2次元DCT ... 信号 $x(n,m)$, 係数 $X(u,v)$

$$X(u,v)=\frac{1}{4}C(u)C(v)\sum_{n=0}^{7}\sum_{m=0}^{7}x(n,m)\cos\left(\frac{(2n+1)u\pi}{16}\right)\cos\left(\frac{(2m+1)v\pi}{16}\right)$$

$$x(n,m)=\frac{1}{4}\sum_{u=0}^{7}\sum_{v=0}^{7}C(u)C(v)X(u,v)\cos\left(\frac{(2n+1)u\pi}{16}\right)\cos\left(\frac{(2m+1)v\pi}{16}\right)$$

ここに,

$$C(u),C(v)=\begin{cases}\dfrac{1}{\sqrt{2}} & (u=0,v=0)\\ 1 & (u\neq 0,v\neq 0)\end{cases}$$

8x8DCT

- 8x8 point 2-D DCT ... Signal *x(n,m)*, Coefficient *X(u,v)*

$$X(u,v)=\frac{1}{4}C(u)C(v)\sum_{n=0}^{7}\sum_{m=0}^{7}x(n,m)\cos\left(\frac{(2n+1)u\pi}{16}\right)\cos\left(\frac{(2m+1)v\pi}{16}\right)$$

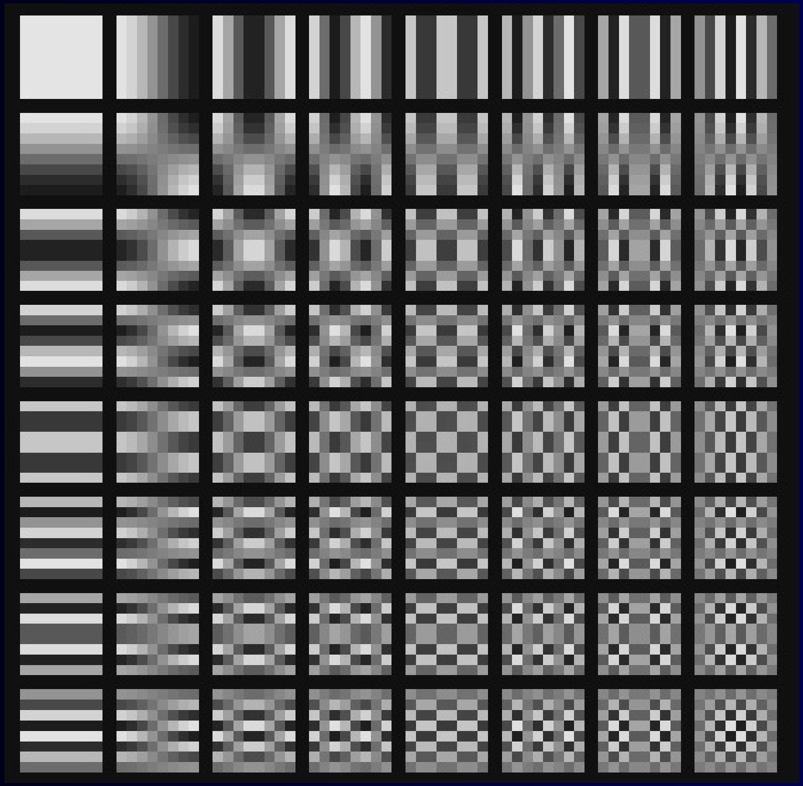
$$x(n,m)=\frac{1}{4}\sum_{u=0}^{7}\sum_{v=0}^{7}C(u)C(v)X(u,v)\cos\left(\frac{(2n+1)u\pi}{16}\right)\cos\left(\frac{(2m+1)v\pi}{16}\right)$$

where,

$$C(u),C(v)=\begin{cases}\frac{1}{\sqrt{2}} & (u=0,v=0)\\ 1 & (u\neq 0,v\neq 0)\end{cases}$$

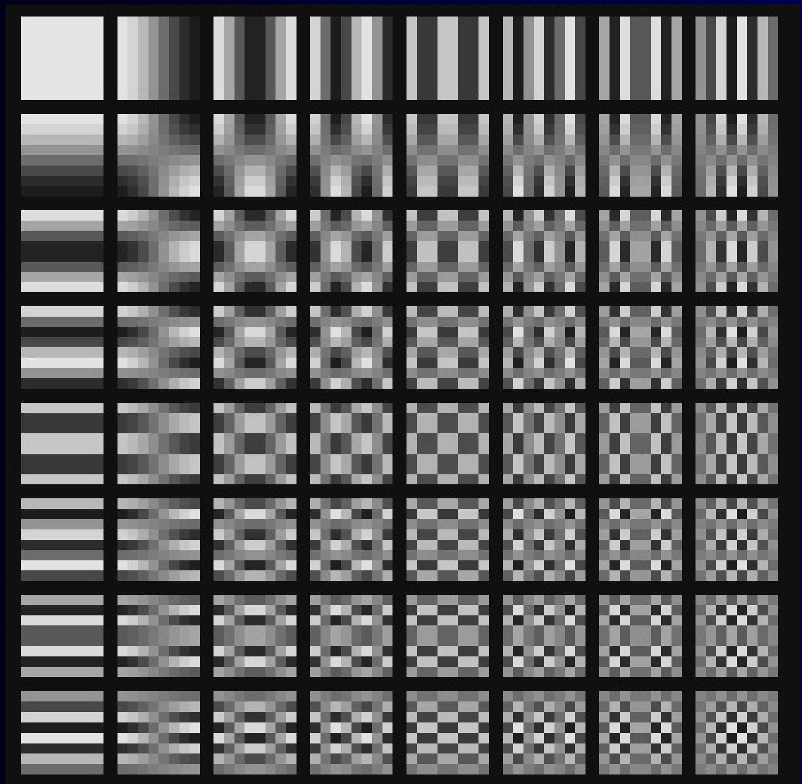
基底画像

- 8x8DCTの基底画像

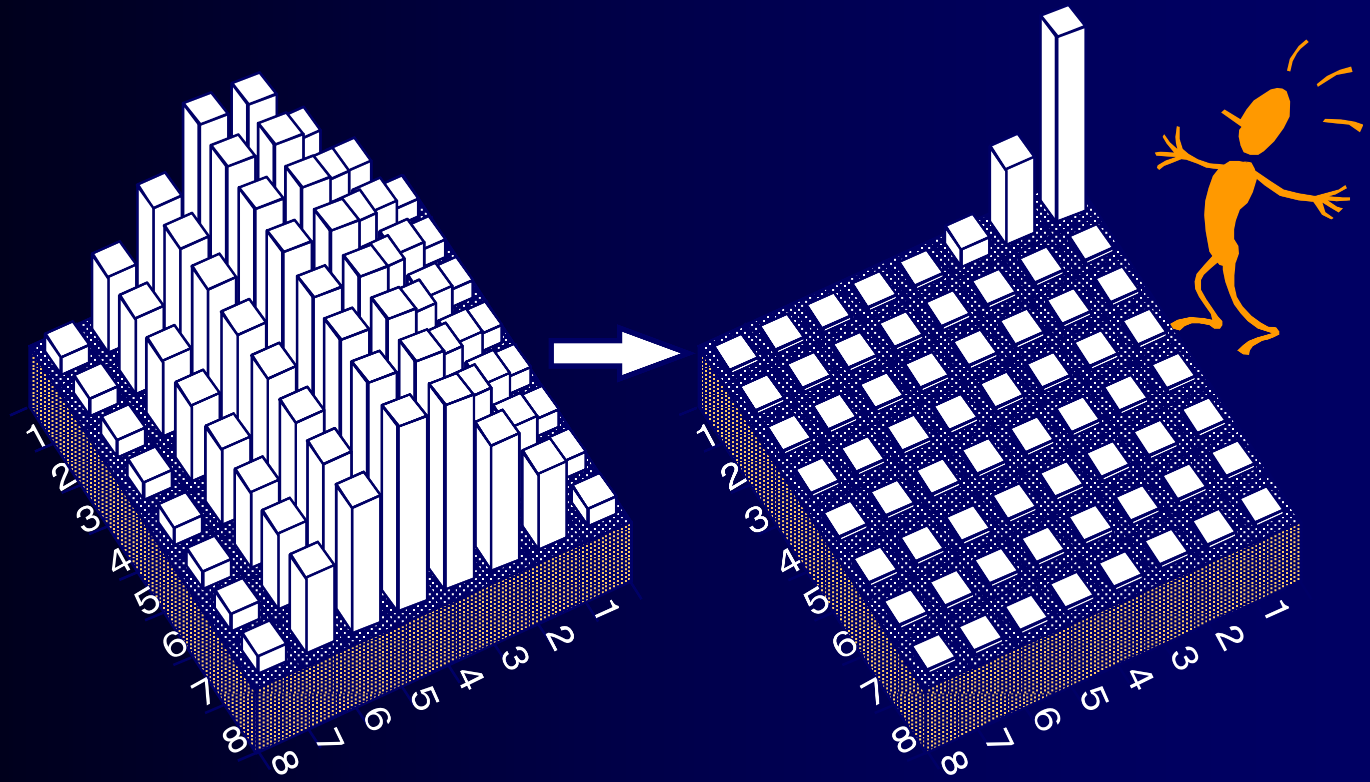


Basis Image

- Basis Image of 8x8DCT

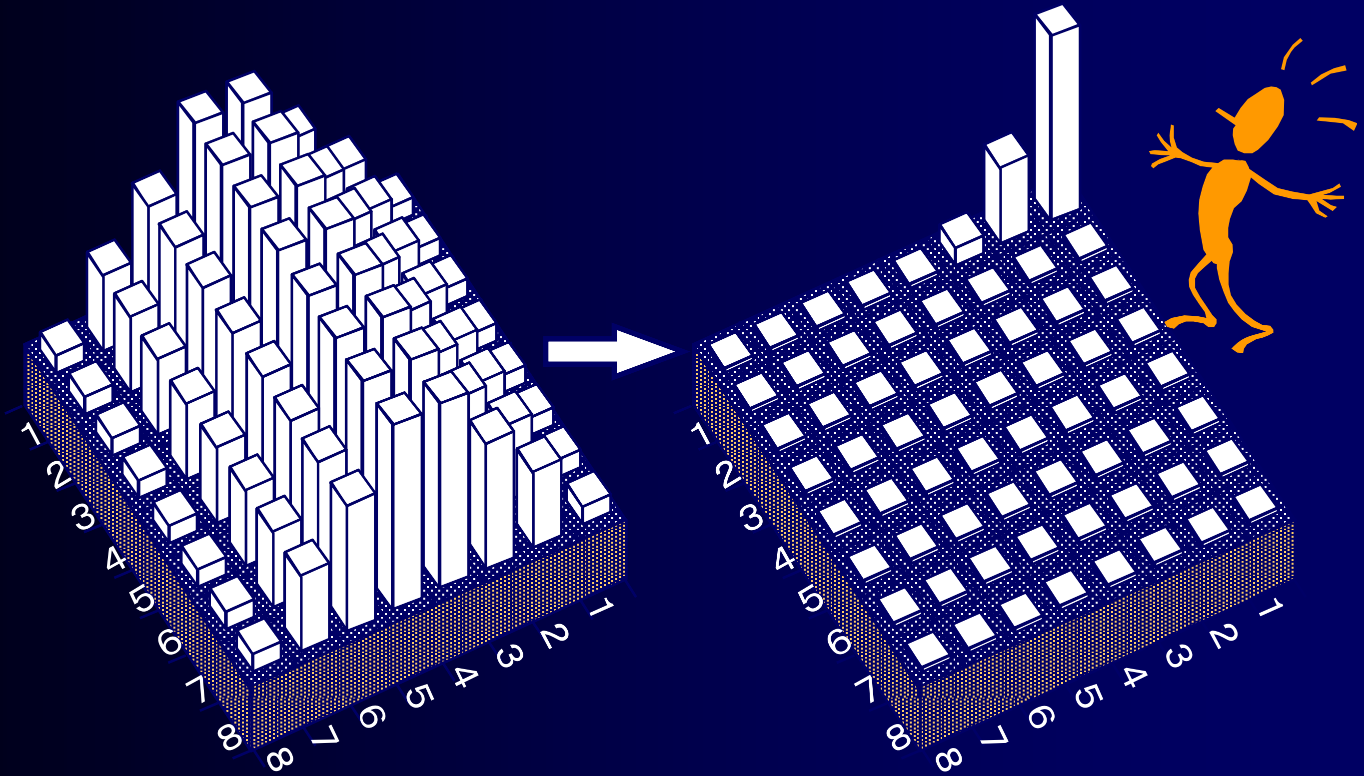


DCTの特徴



信号領域（画素値）　　周波数領域（DCT係数）

Characteristics of DCT



Signal Domain
(Pixels)

Frequency Domain
(DCT Coeffs.)

DCTの種類

- DCT-I

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=0}^{N}C(n)x(n)\cos\left(\frac{nk\pi}{N}\right) \quad (k=0,\ldots,N)$$

$$x(n)=\sqrt{\frac{2}{N}}C(n)\sum_{k=0}^{N}C(k)X(k)\cos\left(\frac{nk\pi}{N}\right) \quad (n=0,\ldots,N)$$

- DCT-II

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=0}^{N-1}x(n)\cos\left(\frac{(2n+1)k\pi}{2N}\right) \quad (k=0,\ldots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{k=0}^{N-1}C(k)X(k)\cos\left(\frac{(2n+1)k\pi}{2N}\right) \quad (n=0,\ldots,N-1)$$

Type of DCT

- DCT-I

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=0}^{N}C(n)x(n)\cos\left(\frac{nk\pi}{N}\right)\quad (k=0,\ldots,N)$$

$$x(n)=\sqrt{\frac{2}{N}}C(n)\sum_{k=0}^{N}C(k)X(k)\cos\left(\frac{nk\pi}{N}\right)\quad (n=0,\ldots,N)$$

- DCT-II

$$X(k)=\sqrt{\frac{2}{N}}C(k)\sum_{n=0}^{N-1}x(n)\cos\left(\frac{(2n+1)k\pi}{2N}\right)\quad (k=0,\ldots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{k=0}^{N-1}C(k)X(k)\cos\left(\frac{(2n+1)k\pi}{2N}\right)\quad (n=0,\ldots,N-1)$$

DCTの種類 (2)

- DCT-III

$$X(k)=\sqrt{\frac{2}{N}}\sum_{n=0}^{N-1}C(n)x(n)\cos\left(\frac{(2k+1)n\pi}{2N}\right)\quad (k=0,\ldots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}C(n)\sum_{k=0}^{N-1}X(k)\cos\left(\frac{(2k+1)n\pi}{2N}\right)\quad (n=0,\ldots,N-1)$$

- DCT-IV

$$X(k)=\sqrt{\frac{2}{N}}\sum_{n=0}^{N-1}x(n)\cos\left(\frac{(2n+1)(2k+1)\pi}{4N}\right)\quad (k=0,\ldots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{k=0}^{N-1}X(k)\cos\left(\frac{(2n+1)(2k+1)\pi}{4N}\right)\quad (n=0,\ldots,N-1)$$

Type of DCT (2)

- DCT-III

$$X(k)=\sqrt{\frac{2}{N}}\sum_{n=0}^{N-1}C(n)x(n)\cos\left(\frac{(2k+1)n\pi}{2N}\right) \quad (k=0,\dots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}C(n)\sum_{k=0}^{N-1}X(k)\cos\left(\frac{(2k+1)n\pi}{2N}\right) \quad (n=0,\dots,N-1)$$

- DCT-IV

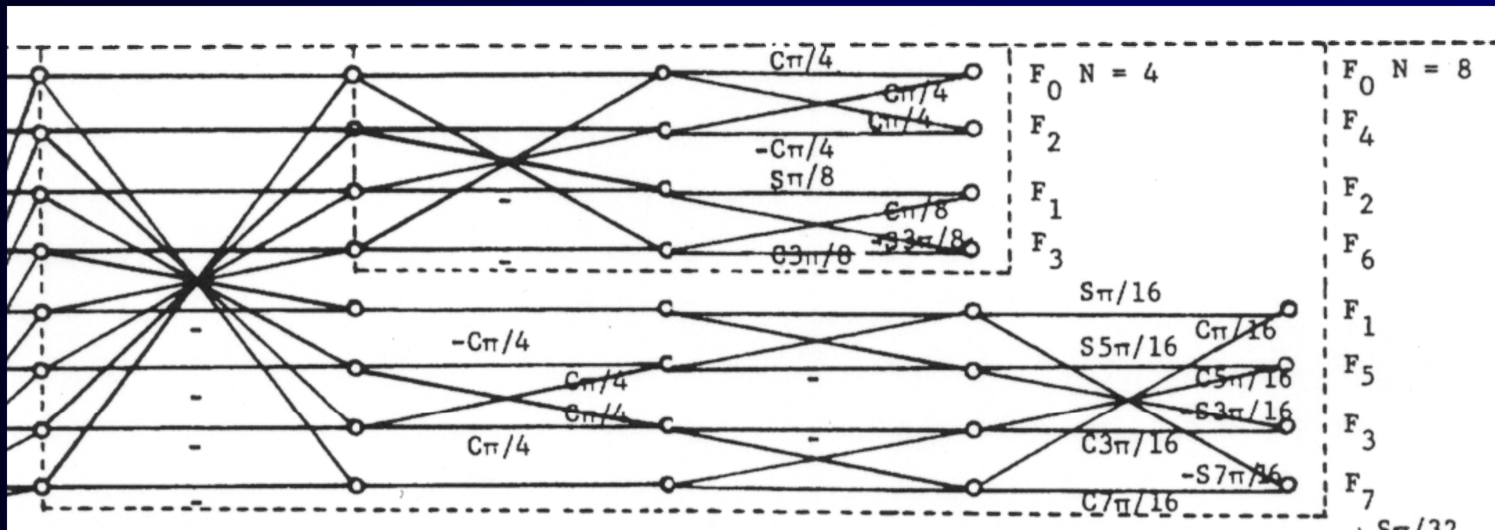
$$X(k)=\sqrt{\frac{2}{N}}\sum_{n=0}^{N-1}x(n)\cos\left(\frac{(2n+1)(2k+1)\pi}{4N}\right) \quad (k=0,\dots,N-1)$$

$$x(n)=\sqrt{\frac{2}{N}}\sum_{k=0}^{N-1}X(k)\cos\left(\frac{(2n+1)(2k+1)\pi}{4N}\right) \quad (n=0,\dots,N-1)$$

DCTの高速算法

- Chen-Smith-Fralick (1977)

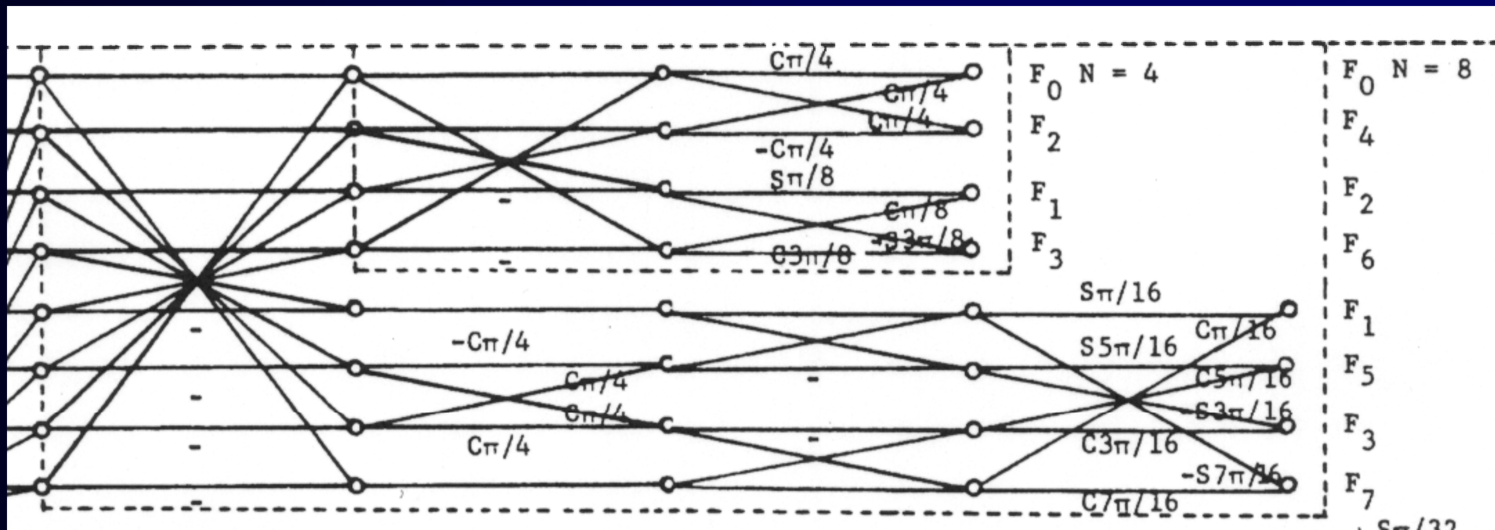
f(0)
f(1)
f(2)
f(3)
f(4)
f(5)
f(6)
f(7)



Fast Algorithm for DCT

- Chen-Smith-Fralick (1977)

f(0)
f(1)
f(2)
f(3)
f(4)
f(5)
f(6)
f(7)



DCTの特徴

- KL変換に近い低域シーケンシーへのパワー集中
- 直流成分は1個の係数で表現できる
- FFTに似たバタフライ演算による高速算法を実現可能
- 画像符号化 (DCT-II)、音響符号化 (M-DCT, MLT)
  - M-DCT: Modified DCT
  - LOT: Lapped Orthogonal Transform
  - MLT: Modulated LOT

Characteristics of DCT

- Energy compaction performance to low sequency is close to KL Transform
- DC component can be represented by one coefficient
- Fast computational algorithm exists like FFT's buttery computation
- Image coding (DCT-II)、Audio coding (M-DCT, MLT)
  - M-DCT: Modified DCT
  - LOT: Lapped Orthogonal Transform
  - MLT: Modulated LOT